Measuring the sound absorbing coefficient in Kundt-tube M11

The properties of various sound-absorbing materials, layers or surfaces are important for acoustic engineers.
There are two typical measurement types considering the geometry of the sound field. It can be random (arriving from every direction), or it can be regular, incident to the tested surface only perpendicularly. The first measurement can be performed in a reverberation room, but the strictly regular, plain waves can be induces in the so called Kundt-tube.

We will use the closed-end variant of the Kundt-tube. The measuring section of the tube is closed by a rigid wall or stopper, and the waves are reflected from it. The two sinusoid waves travelling in the opposite direction interfere and form a standing wave sound field. The absorbing material under test is placed onto the end-wall. It absorbs a part of the incident energy, so the reflected amplitude will be smaller than the incident one. The sound absorbing coefficient can be calculated from the amplitude of the two (incident and reflected) waves.

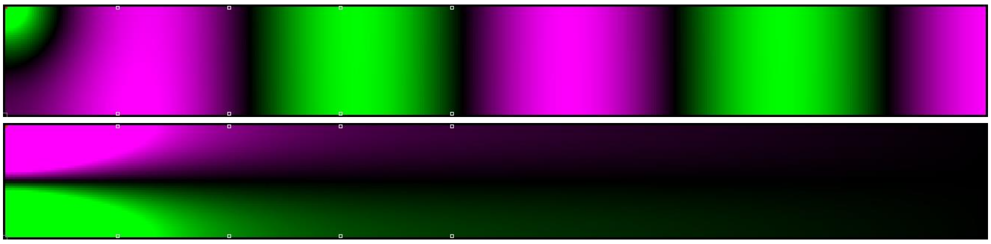
The regularity of the wavefronts in the Kundt-tube is important for two reasons.

- It ensures well-defined measurement circumstances, a well-defined sound field around the absorbing material under test.
- The regular sound field is easier to measure. If the sound pressure is constant along a cross section of the tube, it can be measured by a single measurement in a single microphone position. Otherwise averaging is needed. (inconvenient and time consuming)

Wave propagation in parallel tubes (Or what makes the waves regular in a tube?)

If the air is excited with a loudspeaker at one end of a long, cylindrical tube waves will travel along the tube. If the wavelength is much longer than the diameter of the tube, the waves can be plain waves only. Other forms of wave cannot propagate in the tube at low frequencies. However at higher frequencies really complicated waves with changing, irregular wavefronts can be induced.

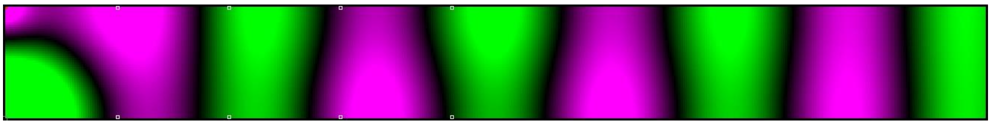
The frequency criterion can be put this way: Let the measuring frequency should be always lower then the first transverse resonance in the tube. This is a theoretical limit; the practical one is lower by 5-10 percent. For explanation let us see the figures below.



f= 0.5*f_{limit}

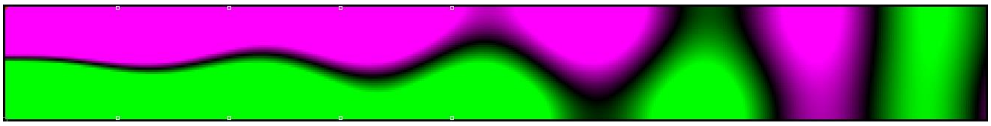
f= 0.5*f_{limit}

Fig. 1. Travelling waves in the Kundt-tube from left to right. At this peaceful low frequency the wavefronts become plain almost immediately. Even if the excitation is one-sided. (top) With antisymmetric excitation however there are no travelling waves. The local movements near the excited end decay fast (exponentially) along the tube. There is no energy transport along the tube. (bottom)



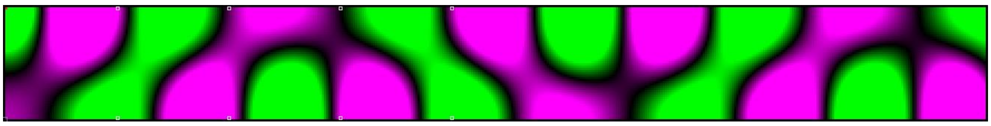
f=0.95*f_{limit}

Fig. 2. Near the cross-resonance the settling of the wavefronts is slower. The disturbing exponential wave-component is also stronger and longer. (one-sided excitation)



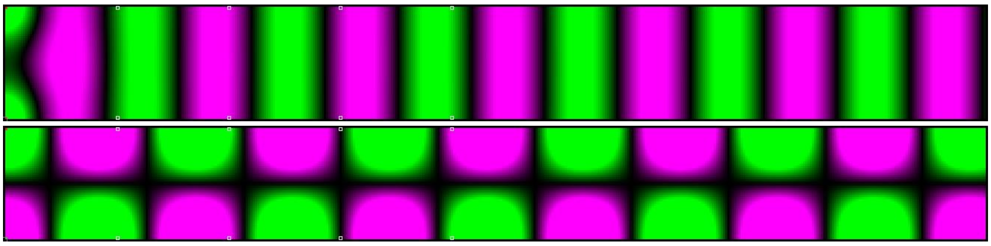
f=0.98*f_{limit}

Fig. 3. The air starts to resonate. At the beginning of the tube the air particles move perpendicular to the tube's axis mainly. (one-sided excitation)



f=1.3*f_{limit}

Fig. 4. At this frequency a new waveform appears too. The new one can reach far; it has no exponential decay with space either.



$f=1.3*f_{limit}$

Fig. 5. These are the two constituents of the combined waves in Fig. 4. They are separated by the different symmetry of excitation. (top: symmetric; bottom: antisymmetric excitation)

If the frequency is low enough, the started waves became regular of their own accord. They need some time (or space) only. For the practice it can be important that how much space, how long tube should be dedicated to this settling. (or how much residual irregularity remains after a given length of travelling).

Wave travelling in tubes — another approach

Let us place a loudspeaker asymmetrically to the beginning of an infinite, cylindrical tube and excite it with slowly rising frequency. First we will find that plain waves can be started at any frequency. But there are more complicated forms of wave that cannot travel under a certain frequency limit. These forms cannot transmit sound energy; they decay along the tube exponentially after leaving the place of birth.

In the tube there are several solutions of the motion equation. The plain wave is valid at every frequency. They are utilized in a Kundt-tube. In its case the pressure varies sinusoidally along the tube but is constant at the perpendicular directions. The particles move parallel to the axis only.

In a tube there exist transverse resonances too. The particles move sideways only, and the pressure does not vary with the length of the tube. Since there is no variation along the tube there is no energy transport either. The wave number along the tube is zero: $k=0$.

In a tube infinite number of transverse resonances is possible. Every one of them has its own frequency. The higher the frequency, the more complicated the pressure pattern is along the cross section.
However if the frequency is increased even a little bit, the particles and the energy start to move parallel to the axis too. The pressure will not be constant along the axis any more; it will vary sinusoidally. It has to be noted that the cross-sectional pressure pattern does not change in the meantime. The wave number along the tube is larger than zero: $k>0$.

Decreasing the frequency below the transverse resonance the cross-sectional pressure pattern remains the same, and the amplitude decreases exponentially along the axis. In this case the wave number is imaginary. There is no energy propagation. The decay-constant $l=i/k$ (the length at which the amplitude drops to 1/e of their original value) varies with frequency. Near the resonance the decay is slow, but further down it becomes faster.

By the loudspeaker every possible waveforms are excited to some extent. If the frequency is chosen well, every not wanted form decays fast and —after a certain distance— the plain wave is left alone.

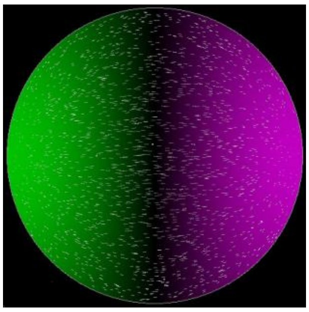


Fig. 6. The first transverse vibration mode of a cylindrical tube. The colors show the pressure, the lines represent the particle velocity. This vibration mode has the lowest frequency; it is the most likely to disturb the measurement in a Kundt-tube.

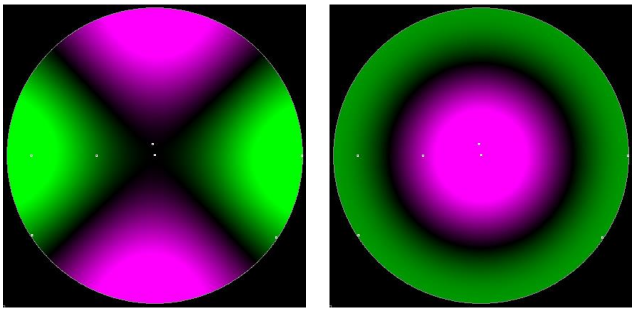


Fig. 7. Two other transverse modes in the tube. Having higher frequency they decay more rapidly at a certain measuring frequency.

After the global behaviour, let us see the concrete numbers...

The (inner) diameter of out Kundt-tube is 103mm. The first transverse resonance is at 1900Hz. Using the Kundt-tube above 1800Hz is not advisable.
At 1800Hz the decay length of the irregularity error is 83mm.[1]

The higher order cross-resonances (Fig. 7.) drop even faster: in 15 and 12mm.

[1] The accurate expression for the decay length is: f $\dfrac{C}{2\pi\sqrt{f_X^2 - f_{meas}^2}}$, where C: sound velocity; f_X: the cross-resonance frequency in question; f_{meas} : the measuring frequency.

The measuring section of our Kundt-tube is 1 meter long. The asymmetrically placed loudspeaker is 15cm far from the measuring section. At 1800Hz the wavelength is 17cm. The movable microphone on the microphone rod or slider (Fig. 8.) moves in the region of 3-4 quarter wavelengths near the closed end. There remains about 80cm length for the wavefronts to smooth. It means that the pressure unevenness will decrease to the $e^{(-800/83)}$ = 67 millionth of the original value.
At 100Hz (the bottom of the frequency range) the quarter wavelength is 84cm. The two possible microphone positions are near the wall (loop), and about 84cm from it (node). At 100Hz the unevenness drops to 1/e at every 23mm along the tube. It seems to be fast enough, especially, if the original unevenness was not too serious. (At a low frequency it is almost impossible to make irregular waves with a small loudspeaker.)

After examining the upper and lower end of the frequency range, it appears that the wavefronts in the tube can be almost perfect. Measurement errors (if exist :-) will likely be caused by other sort of disturbances.

Measurement with the Kundt-tube

The air column in the Kundt-tube is excited at one end, waves are sent toward the other end. They are partially reflected at the closed end, and coming back the two waves form a standing wave sound field. The amplitude of the returning waves depends on the absorbing coefficient of the end. The sound field can be scanned by the movable microphone. There is another microphone too — built into the end-wall of the tube. This microphone provides for a reference signal for the data processing; the pressure at the end-wall is always proportional to the incident wave's amplitude.

At the excited end around the loudspeaker a lot of sponge is stuffed into the tube to diminish the longitudinal resonances of the air column. It is useful especially when measuring thin absorbing materials or the empty tube. Too sharp resonances would be harmful; the tube would behave very immoderately.

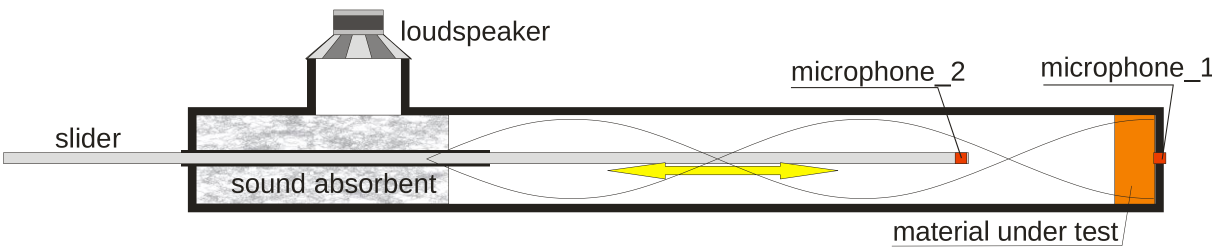


Fig. 8. The sketch of our Kundt-tube. With a microphone at the tip of the slider rod, the sound field can be scanned. Another microphone is built into the back-wall to produce a reference signal.

The sound absorbing coefficient of a sponge sample will be measured, in case the sample is placed on a rigid wall. To achieve this, the ratio should be determined between the incident and the reflected amplitudes at various frequencies.

The standing wave ratio

When two sinusoid waves of the same amplitude go in opposite direction the resulted standing wave is perfect. In this case there is no energy propagation in the tube (only a small oscillation within every quarter wavelength). Scanning this standing wave with a microphone, very deep minimum places can be found. The more accurately identical the two amplitudes, the more deep these minimums are.

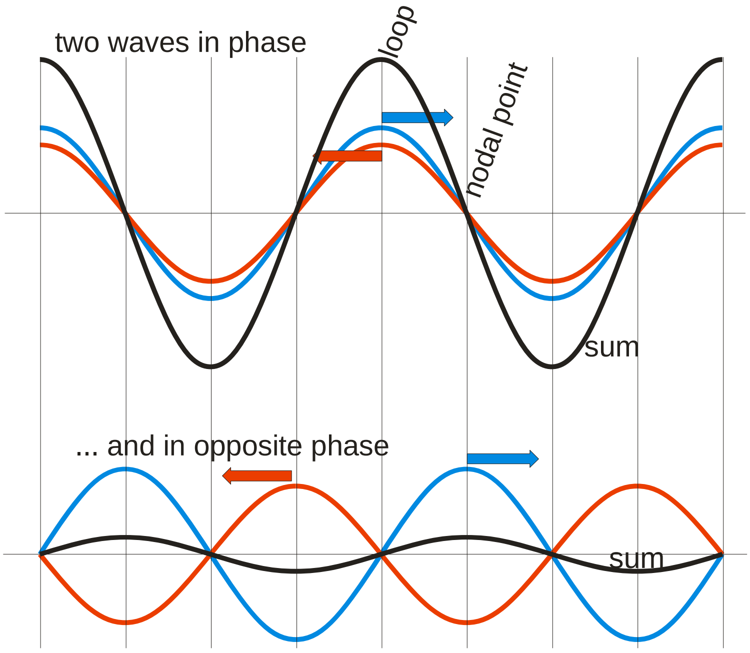


Fig. 9. Two waves (red and blue) travel in opposite direction. They are just in phase (top); they are in opposite phase after a quarter time period (bottom).

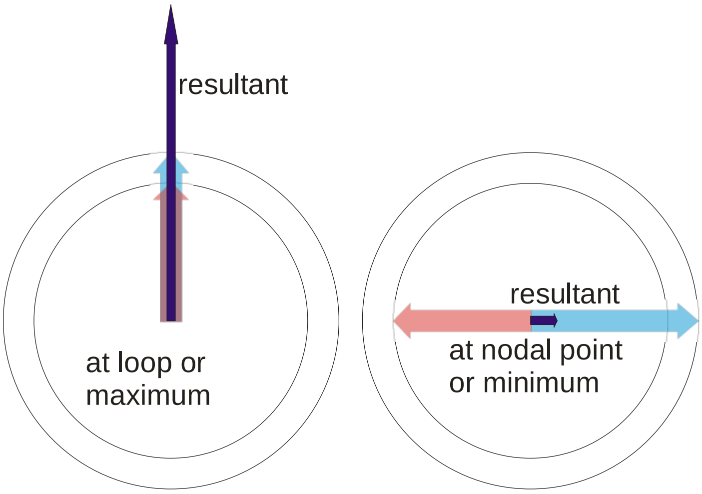


Fig. 10. Complex amplitudes of the two travelling waves. The phase at the minimum or nodal point is perpendicular to the phase at loop.

When the two sinusoids are just in phase, the sum of their amplitudes can be measured at the maximum point. After a time of quarter period they will be in opposite phase. Now the difference can be measured a quarter wavelength apart from then maximum point. (This is the nodal point.)
With the amplitudes of the incident and reflected waves can be written (p_{max} is the pressure amplitude in a loop; p_{min} is the same for a node.)

$$p_{max} = p_{incident} + p_{reflected}$$

and

$$p_{min} = p_{incident} - p_{reflected}$$

After a rearrangement:

$$p_{incident} = \frac{p_{max} + p_{min}}{2}$$

$$p_{reflected} = \frac{p_{max} - p_{min}}{2}$$

At a loop the pressure shows a relatively flat maximum, and the phase is stable too; they do not vary with place too fast.
However, measuring the pressure accurately at a sharp node it is more difficult. Both the amplitude and the phase is strongly space-dependant. The problems can be reduced by proper signal processing. (see Fig. 11.)
The measurement should be phase-correct. That is, the signal from the moving microphone should always be referred to the fixed one. “Referred to” means “divided by”. This division results in two advantages.

1. The phase of a “bare” microphone signal is random. It depends on how the sine falls into the time record. However, the phase difference between two microphone signals is a stable value, depending on the microphone position(s) only.
2. The loop and the node pressure will be measured by the same microphone, so it has to be carried out in different moments. That is, the entire sound field can alter in the meantime. This change (if not corrected) causes a measurement error. The automatic $p_{slider} / p_{reference}$ division makes the needed correction.

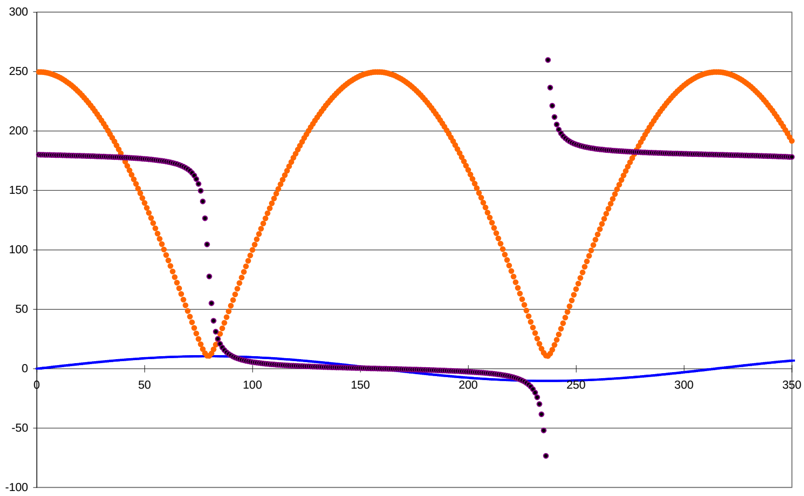


Fig. 11. Typical space functions along the Kundt-tube.
red: Absolute value of the pressure.
black: The phase (relative to the reference signal) in degrees. It changes at the nodes very rapidly, while at the loops it is relatively stable.
blue: The vector component of the complex amplitude perpendicular to the phase at loop.

We are interested in the ratio of the incident to the reflected sound energy. The energy absorbing capability of the measuring end of the tube is:

$$\alpha = \frac{E_{dissipated}}{E_{incident}} = \frac{E_{incident} - E_{reflected}}{E_{incident}}$$

The transported energy is always proportional to the square of the amplitude. The absolute measure of the energies (or powers) has no importance; α depends on ratios only.

$$\alpha = \frac{p^2_{incident} - p^2_{reflected}}{p^2_{incident}}$$

Using a previous expression for $p_{incident}$ and $p_{reflected}$ it can be written, that

$$\alpha = \frac{(p_{max} + p_{min})^2 - (p_{max} - p_{min})^2}{(p_{max} + p_{min})^2}$$

After a simplification

$$\alpha = \frac{4 p_{max} p_{min}}{(p_{max} + p_{min})^2}$$

It is practical to use the $R = p_{min} / p_{max}$ substitution. It emphasizes that α depends on the ratio of pressures.

$$\alpha = \frac{4R}{(R+1)^2}$$

Scanning the pressure amplitude along the tube

The slider can be moved up and down to scan the pressure along the tube. First of all it is advisable to get some experience with it. A frequency of 1200Hz is suggested - with sound absorber in the tube. It will show the phenomena clearly. While scanning, check the various quantities measured and displayed.

Calibration

Since the absorption depends on a ratio, calibration of the microphones is not needed. However, the linear behaviour of the entire measuring channels is required.

Choosing the correct pressure amplitude

There are two considerations.

1. If the pressure is too high, the loudspeaker or measuring channel can be overloaded. The loudspeaker's distortion does not result in an error, because the applied frequency analysis gets rid of the harmonics. The loudspeaker, however, does not like to be tortured.
2. With too small amplitude the signal can sink into the noise. Noise can come from the measuring system itself or from the environment as well.

Practice shows, that generally a wide amplitude range can be used. This is the advantage of the frequency analysis as follows.

Data processing

The signals from the microphones contain disturbing components as well. (noise, hum, nearby conversation). If using frequency analysis, these signal parts have small affect on the measurement results only. The explanation is that the frequency of the noise differs from the one of the measurement.)

The following method makes it possible to pick a sinusoid component from a noisy signal. It delivers both the amplitude and the starting phase.

- A certain (e.g. 1 sec long) signal is recorded.
- It is multiplied by a Hanning window to smooth the sharp cuts at the ends. (it is advantageous to the selectivity)
- Two time series are calculated too with the measuring frequency. (a sine and a cosine)
- The Hanning-ed input signal is multiplied by them and summed up. The two results are the real and imaginary parts of the complex amplitude.

During the measurement the slider-microphone's signal amplitude is always divided by the reference signal's. (We do not have to be afraid of the nominator becoming too small. At the closed end of a tube the sound pressure is always high — even if the microphone is covered by a sponge layer.)

Error sources

- There can be crosstalk between the microphone channels or with the loudspeaker signal. (Checked. They are small.)
- The tube's wall is not ideally rigid. The vibrating wall can give or receive energy from the air. Probably this is the main problem with our Kundt-tube. The tube should have thick wall of steel, and be dampened somehow too.

At certain frequencies we can get contradictious results. The reason is likely the vibration of the tube's wall or the stopper.

The measurement setup

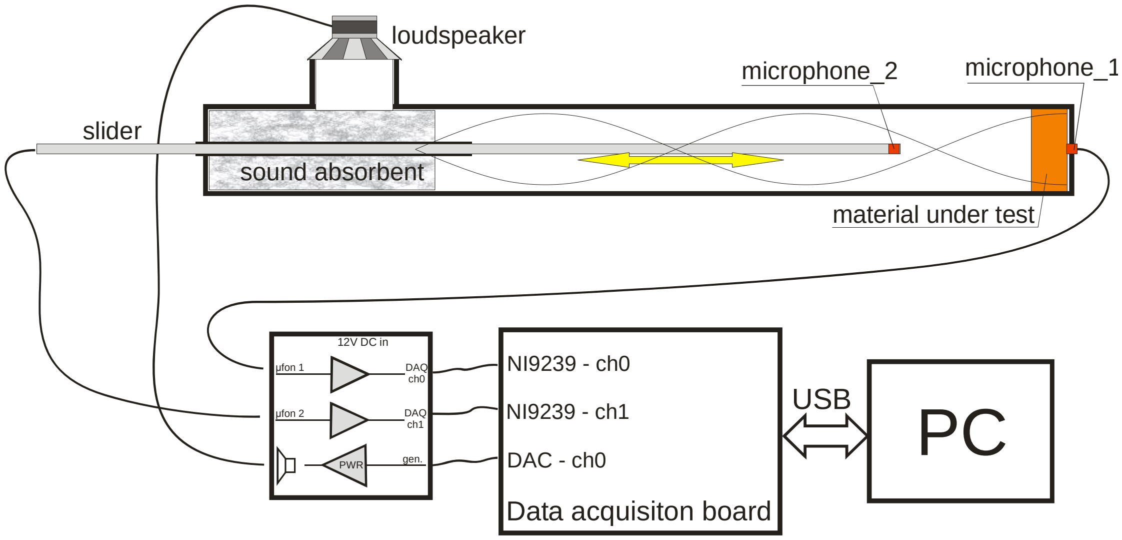


Fig. 12. The sketch of the total measurement setup.
The analogue output from the data acquisition board (DAQ) is amplified and fed into the loudspeaker. The microphone signals are sampled.

Execution of the measurement

- assembling the measurement system
- placing the absorbing material (sponge) into the tube
- putting back the end-wall stopper as well (Between the wall and the sponge, should not be too much gap. It is advisable pushing the sponge to its place by the stopper.)
- starting the measurement program
- choosing a frequency between 100Hz and 1800Hz
- finding a correct amplitude
- finding a loop by moving the slider
- noting the amplitude (or pushing „set loop (ref.)" button in the control window)
- finding a nodal point
- noting it or simply reading the calculated sound absorbing ratio, calculated automatically and displayed in the window

The complete measurement should be performed at 10 frequencies in the usable frequency range.
The measurement should be repeated with empty tube too. The final result is the difference between the "dampened" and the "empty" results.

The measurement report should contain the short description of the measurement and the results.

If you have enough time spared, the measurement can be carried out with two layers of sponge.
Near the wall the pressure amplitude is always high but the particle velocity is small. It can be interesting to measure the efficiency if the sponge is placed at velocity maximum, $\lambda/4$ apart from the wall. (This measurement is slower, because λ is frequency-dependant; the sponge always has to be moved on and on.)

Hints:
At high frequencies many loops and nodes fit into the tube. It makes possible to perform more then one measurement at one frequency. From the difference between the results, we can try to estimate the measurement accuracy.

Below 200Hz only one node can be found in the tube. A point, close to the sponge can be used as loop. This is a good approximation at large wavelength.

Appendix 1
Ingredients of the measuring system

Kundt-tube
- microphone rod or slider
- white plastic „end-wall" or stopper , with the reference microphone built into it

3 sponge samples
Amplifier unit to handle the microphone signals and drive the loudspeaker
12V DC power supply
4 BNC-BNC cables
National Instruments data acquisition board
- NI 9263 signal generator. 100kHz, 4 channels (one of them is used)
- NI 9215 analog input (=AD converter) 100kHz, ±10V, 16bit, 4 channels with synchronous sampling (two of them are used)
- NI cDAQ 9174 "motherboard"
- NI power supply

Laptop + power supply
Measuring software

Fig. 13. (left): The plastic end-wall or stopper with the microphone in the middle, sealed by a black O-ring; A sponge sample; The open end of the tube with the tip of the microphone-slider;
(right): The excited end of the a Kundt-tube (the loudspeaker is placed behind the orange stopper) ; The microphone-slider (a brass rod) and the amplifier unit can be seen too

Appendix 2
Handling the measurement software

(An instructor will be present to help the first steps.)

The software can be started by the shortcut **pulse_root.exe** on the table.

The AD converters work continuously at a high frequency (100kHz) and sample the microphone signals. Digital filters cut off the too high frequencies and the data stream is resampled. (This is the so called decimation.) The processes working behind the window named “Kundt – tube” or “Kundt cső” use this slower data stream. (Fig. 14.)

The algorithm regularly picks up a section from the continuous data stream to determine the complex amplitude of the microphone signals. From these two amplitudes various quantities are calculated and displayed.

The measuring frequency can be adjusted in the window named “sound generator”, using the control fields. (Fig. 16.)

Working with blue numeric control fields
Clicking on it (left-right) the number will step (up-down) roughly.
Clicking with *ctrl* or *alt* or *ctrl+alt* the stepping will be more and more fine.
Clicking with *shift* the contents can be edited “by hand”.
The numbers are displayed with using the prefixes: n, μ, m, k M, G and so on. For example 123m = 123milli = 123E-3. (It is shorter and easier to read the quantities this way.)

In Fig. 15. the main control windows of the application can be seen. Closing this, the program stops.

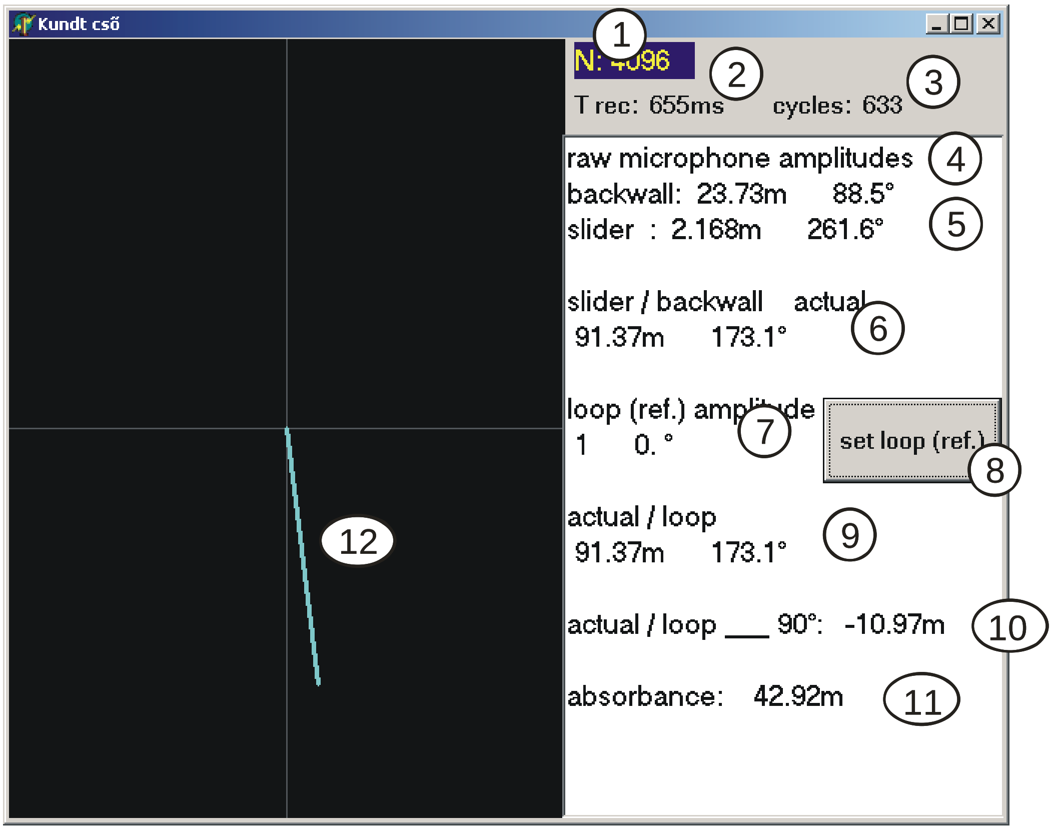


Fig. 14. The special window required for the Kundt-tube measurement.

The Kundt-tube window and the displayed quantities
1) The number of samples in one measurement (or record)
2) The length of it expressed in seconds
3) The number of sine periods in one record
4), 5) the amplitude and phase of the back-wall and slider signals
These phases are practically random; the amplitudes are sensitive to the excitation.
6) The complex ratio of the two previous amplitudes. The phase is stable and the amplitude does not vary with the excitation amplitude too much.
7), 8) The complex amplitude (at the loop) is stored here if the button is pushed. Every quantity below this line will be referred to this loop amplitude.
9) The actual slider amplitude (relative to the one at the loop)
10) The same but the perpendicular component only
11) The sound absorbing coefficient
12) The (9) vector displayed graphically. (Becoming too small, it changes range and the color too. It is a method to increase the useful amplitude range of the display.)

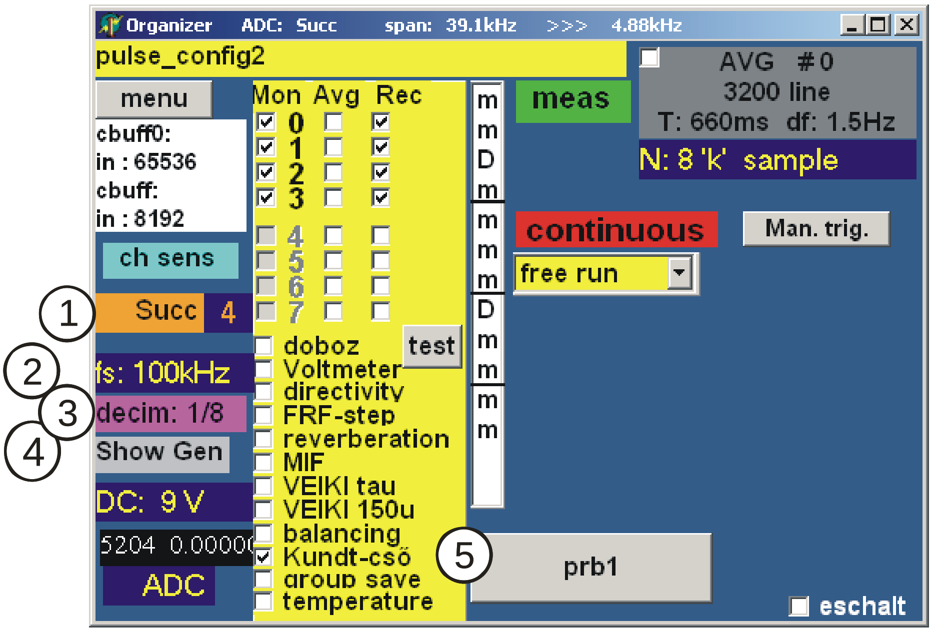


Fig. 15. The main window of the measuring system. The following parts are important for us.
1) The used AD converter
2) The primary sampling frequency (100kHz is correct)
3) Decimation factor of the data stream (1/8 is OK)
4) If the “sound generator” window is hidden it can be displayed with this button.
5) At the “Kundt-cső” should be a “check”. Otherwise the window does not appear.

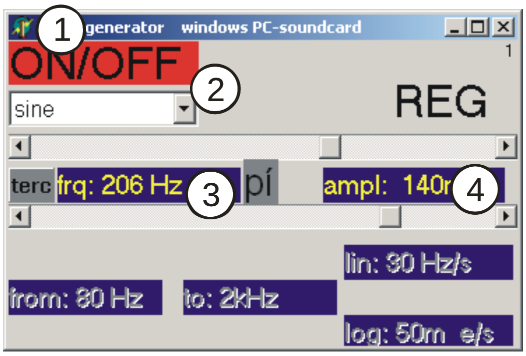


Fig. 16. The sound generator.
1) ON-OFF switch. (click on it)
2) Signal form
3) Frequency
4) Amplitude; the possible range is 0...1
(There is no need to go higher than 200m (=0.2).)