1st Problem:

We want to calculate the pressure force which acts on a plane. The plane can be divided into three sub surfaces. We can measure their areas and the forces act on these sub-planes. The measured area, pressure values and also the measurement uncertainties can be found in the first table. Calculate the pressure force which acts on the plane and the uncertainty („error")!

	area		**pressure**	
i	A_i		p_i	
	[cm²]		[Pa]	
1	50.0±0.3		152.25±1.5	
2	24.0±0.2		137.61±1.4	
3	36.0±0.25		118.98±1.2	

The problem can be solved by the following way. The pressure force:

$$\mathrm{F} = \sum_{i=1}^{3} \mathrm{p_i} \cdot \mathrm{A_i} = \mathrm{p_1} \cdot \mathrm{A_1} + \mathrm{p_2} \cdot \mathrm{A_2} + \mathrm{p_3} \cdot \mathrm{A_3} = 15198.42\,\mathrm{Pa} \cdot \mathrm{cm}^2 = 1.519842\,\mathrm{N}$$

Actually, there is no need to display numbers with many digits after the decimal point; the result's accuracy will be determined based on the error calculation.

The result depends on six, independently measured variables:

$$F = F(A_1, A_2, A_3, p_1, p_2, p_3).$$

The result's sensitivity to changing the A_1 input parameter is described by the following *sensitivity coefficient*:

$$\frac{\partial F}{\partial A_1} = p_1 \approx 150\,\mathrm{N/m^2}$$

With the sensitivity coefficient we can calculate the result's uncertainty. Moreover, the measurement uncertainty δA_1 is needed. It comes from the A_1 input parameter's measurement error:

$$\frac{\partial \mathrm{F}}{\partial \mathrm{A_1}} \cdot \delta \mathrm{A_1} \approx \left(150\,\mathrm{N/m^2}\right) \cdot \left(0.3\,\mathrm{cm^2}\right) = 4.5 \cdot 10^{-3}\,\mathrm{N}$$

See the following table for the sensitivity coefficients and uncertainty components which belong to the input parameters given on the left. (Note: There is no need to display numbers with many digits after the decimal point, two is enough.)

input parameter	sensitivity coefficient	uncertainty component
A_1	$\frac{\partial F}{\partial A_1} = p_1$	$\frac{\partial \mathrm{F}}{\partial \mathrm{A_1}} \cdot \delta \mathrm{A_1} \approx 4.5\,\mathrm{mN}$
A_2	$\frac{\partial F}{\partial A_2} = p_2$	$\frac{\partial \mathrm{F}}{\partial \mathrm{A_2}} \cdot \delta \mathrm{A_2} \approx 2.8\,\mathrm{mN}$
A_3	$\frac{\partial F}{\partial A_3} = p_3$	$\frac{\partial \mathrm{F}}{\partial \mathrm{A_3}} \cdot \delta \mathrm{A_3} \approx 3.0\,\mathrm{mN}$
p_1	$\frac{\partial F}{\partial p_1} = A_1$	$\frac{\partial \mathrm{F}}{\partial \mathrm{p_1}} \cdot \delta \mathrm{p_1} \approx 7.5\,\mathrm{mN}$
p_2	$\frac{\partial F}{\partial p_2} = A_2$	$\frac{\partial \mathrm{F}}{\partial \mathrm{p_2}} \cdot \delta \mathrm{p_2} \approx 3.4\,\mathrm{mN}$
p_3	$\frac{\partial F}{\partial p_3} = A_3$	$\frac{\partial \mathrm{F}}{\partial \mathrm{p_3}} \cdot \delta \mathrm{p_3} \approx 4.3\,\mathrm{mN}$

As can be seen, the highest uncertainty component comes from the p_1 parameter's measurement error. The result's uncertainty comes from the sum of squares of the independently measured variable uncertainties:

$$(\delta F)^2 = \sum_{i=1}^{3}\left(\frac{\partial F}{\partial A_i}\cdot\delta A_i\right)^2 + \sum_{i=1}^{3}\left(\frac{\partial F}{\partial p_i}\cdot\delta p_i\right)^2 = \left(p_1\cdot\delta A_1\right)^2 + \left(p_2\cdot\delta A_2\right)^2 + \left(p_3\cdot\delta A_3\right)^2 + \left(A_1\cdot\delta p_1\right)^2 + \left(A_2\cdot\delta p_2\right)^2 + \left(A_3\cdot\delta p_3\right)^2$$

$$\delta F \approx 11\,\mathrm{mN}$$

The resultant force, taking into consideration the measurement uncertainties:

$$\mathrm{F} = (1.52 \pm 0.01)\,\mathrm{N}$$

The uncertainty in the measurement result defines how many decimal places you should report. In this case, the uncertainty in the result is in the second decimal place, so the measurement result should be stated to two decimal places.