List of example problems for BSc laboratory measurements

1. Describe the calculation of air density based on atmospheric pressure and temperature!
Give the names and units of the variables in the formula!

To determine air density, we use the ideal gas law. This is rearranged to arrive at the expression for density:

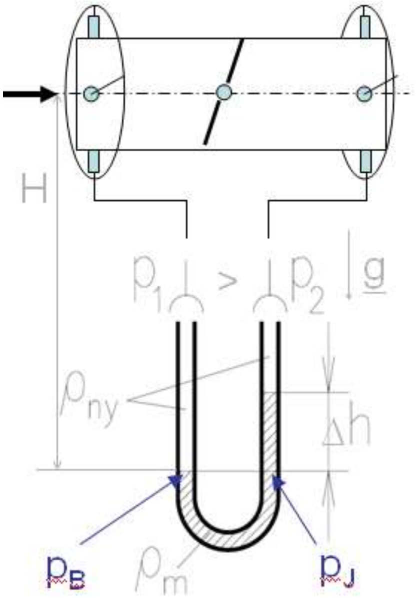
$$\boxed{\rho = \frac{p_0}{RT}}$$, where:

ρ: air density [kg/m^3]
p_0: atmospheric pressure [Pa]
R: specific gas constant of air [J/kg/K];
T: air temperature [K]

2. Describe the U-tube manometer in a few sentences! Provide the equation for calculating the pressure difference from the level difference if the U-tube is filled with a measurement fluid of density ρ_m, and it is connected to a horizontal pipe. Water flows in the pipe, and a butterfly valve causes pressure loss between the two measurement points (hint: draw a figure).
Give the names and units of all the variables in the equations!

In the **U-shaped glass tube** of the manometer, there is a measurement fluid (ρ_m) that **does not mix** with the fluid flow (density ρ_{ny}), but has a **higher density**. This is most often mercury (if water is the flowing fluid) or water, or alcohol (if the flowing fluid is gaseous).

The balance equation of the manometer is written for the lower interface between the fluids, where pressures in the two branches (p_B, p_J) are equal:



$$\boxed{p_B = p_J}$$

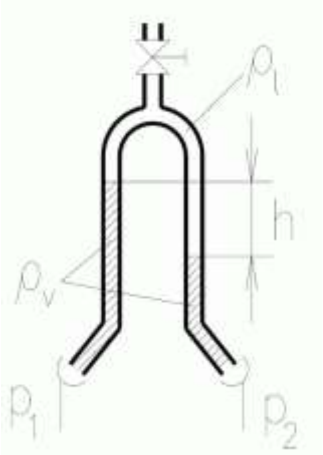
$$p_1 + \rho_{ny} g H = p_2 + \rho_{ny} g (H - \Delta h) + \rho_m g \Delta h$$

$$\boxed{p_1 - p_2 = (\rho_m - \rho_{ny}) g \Delta h}$$

$$\boxed{\Delta p = (\rho_m - \rho_{ny}) g \Delta h}$$

3. How does an upside-down U-tube manometer work, and when do we use it? Calculate the pressure difference from the level difference between the two legs of the manometer!

Application:



Upside-down U-tube manometers measure the **pressure differences** in pipes transporting **liquid media** (water, oil). The upside-down U-tube manometer is a tube closed at the top, in which a gas section above the liquid (usually air) connects the pressure above the liquid rising in the two legs. Thus, a pressure that can be considered the same develops above the liquid in both stems.

Its advantage is that smaller pressure differences can be measured with it much **more accurately** than, for example, with a conventional mercury-filled U-tube manometer, and therefore the **relative error of the measurement is significantly reduced**.

Since the density of the flowing fluid (usually water) and the gas at the top (usually air) differ by several orders of magnitude, we typically consider only the density of water when calculating the pressure difference.

Calculations with an upside-down U-tube manometer:

$$\boxed{\Delta p = (\rho_v - \rho_l) \cdot g \cdot h} \qquad \boxed{\rho_v >> \rho_l} \qquad \boxed{\Delta p = \rho_v \cdot g \cdot h}$$

4. How can one measure the static pressure of a medium flowing in a pipe? How can the necessary design specifications be justified? What equations can be used to support this?

When measuring static pressure, it is a basic requirement that the pressure taps should **NOT affect the flow**, for example, it should not deflect the streamlines, so the inside of the pipe wall **cannot be burred** at the hole, and the inner edge of the hole **should not be significantly chamfered**.

Measurement:

A **pressure tap** with a hole diameter of 0.5-1 mm is made on the wall of the pipe, then a pressure outlet pipe is attached to the outer surface of the pipe wall around the hole in an **"air-tight"** manner. We connect the **pressure transmission tube** (e.g. silicone tube) to this outlet pipe, which connects the hole to the **manometer**. The pipe must be filled entirely by the flowing medium, e.g. in the case of water, **no air bubbles can remain** in the pressure transmission pipe.

Reasoning:

The normal component of the Euler equation, written in the natural coordinate system for steady flow and neglecting the gravitational field of force: $\boxed{\frac{v^2}{R} = \frac{1}{\rho} \cdot \frac{\partial p}{\partial n}}$, where

v: the fluid velocity [m/s]

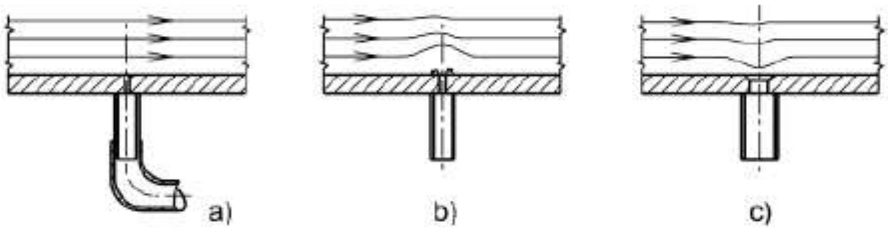
R: the radius of the curvature of the streamline [m]

ρ: density of fluid medium [kg/m^3]

$\boxed{\frac{\partial p}{\partial n}}$: the gradient of the pressure change [Pa/m]

It can be seen from the equation that if the streamlines are curved, a pressure gradient develops perpendicular to the streamlines in the normal direction. If the streamlines are straight, the radius of curvature is R= ∞, and there is no pressure change perpendicular to the streamlines!

Therefore, the appropriate design is case a), as case b) results in a lower and case c) results in a higher pressure on the pressure tap than what would be measured in the flow.



5. Describe the EMB-001 hand-held digital manometer! Provide the functions of the most important buttons and the distribution of the pressure ports!

The **digital manometer** is a **two-channel** digital measuring device that works on the piezoelectric principle and contains **two highly sensitive built-in pressure transmitters**. **The device can measure pressure within the range of $\Delta p = \pm 1250$ Pa with an uncertainty of 2 Pa**. With the device, it is possible to **continuously record the measured values on a computer** via a USB port.

Above the display, channel 1 is measured on the ports to the right, and channel 2 on the ports to the left. If we measure relative to atmospheric pressure, overpressure must be connected to the (top) connectors closer to the display and depression to the bottom connectors in order for positive values to appear on the display.

Functions of the most important buttons:

I/O: ON/OFF switch

0 Pa: zeroing, specifying a zero pressure difference

CH I/II: channel switch. The number on the right side of the display indicates the measured channel

Fast/Slow: change averaging time. The letter on the right side of the display indicates the averaging time (**F**ast/**M**edium/**S**low)

A mérési tartomány: $\Delta p = \pm 1250 Pa$

A mérési hiba: $\delta\Delta p = 2 Pa$

6. Describe static, dynamic, and total pressure (where there is one, the relation between them in an equation form, the definitions, and the units of measurement of the quantities included in the equation), as well as the method of their measurement!

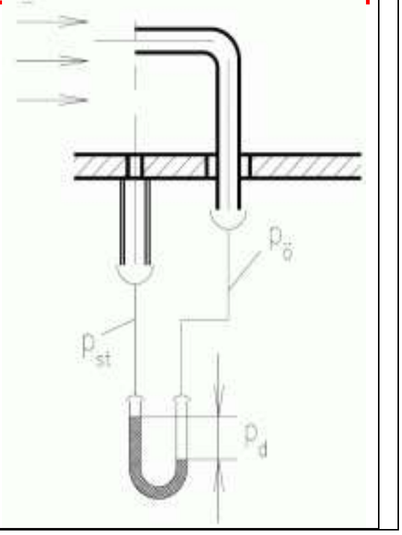
$$p_\infty + \frac{\rho}{2} v_\infty^2 = p_t = p_{\ddot{o}}.$$

Static pressure: pressure in the undisturbed flow. Symbol: p_∞ p_{st} [Pa]

Total pressure: a stagnation point pressure (pressure of the stopped fluid), symbol: p_t, [Pa]

Dynamic pressure: the difference of the two above: $p_d = \frac{\rho}{2} v_\infty{}^2$, where v_∞ is the undisturbed flow velocity, ρ is the density, [Pa]

Static pressure is measured using **static pressure taps**, total pressure is measured with a **Pitot probe**, and dynamic pressure is measured as **the difference between the two**. In this case, the basic requirement is that the pressure not change perpendicular to the flow direction, **the streamlines cannot be curved**. Otherwise, a **Pitot-static probe** (Prandtl probe) should be used.

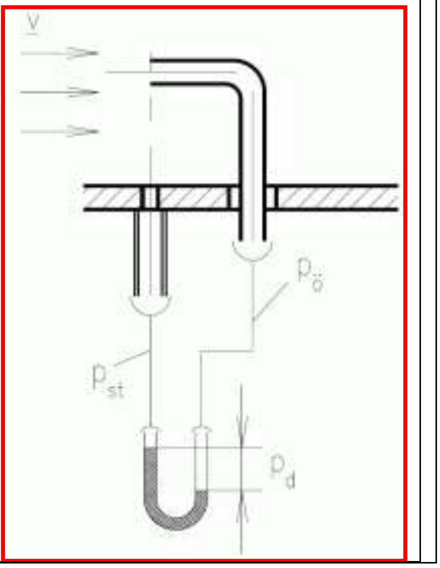


7. Describe the measurement of velocity using a Pitot probe. Illustrate your explanation with a sketch!

The Pitot probe is a pipe turned into the flow, with which the pressure (**total pressure**) of the stopped medium can be measured if we prevent the medium from flowing in the Pitot tube (e.g., we close off the end of the pipe with a manometer). If the streamlines in the examined flow are **straight and parallel**, we can measure the **static pressure** on the wall using a pressure tap. The **dynamic pressure** is the difference between the total pressure and the static pressure, from which, knowing the density of the medium, the flow velocity can be determined:

$$\Delta p = p_d = p_{\ddot{o}} - p_{st};$$

$$v = \sqrt{\frac{2 \cdot p_d}{\rho}} = \sqrt{\frac{2 \cdot \Delta p}{\rho}}$$

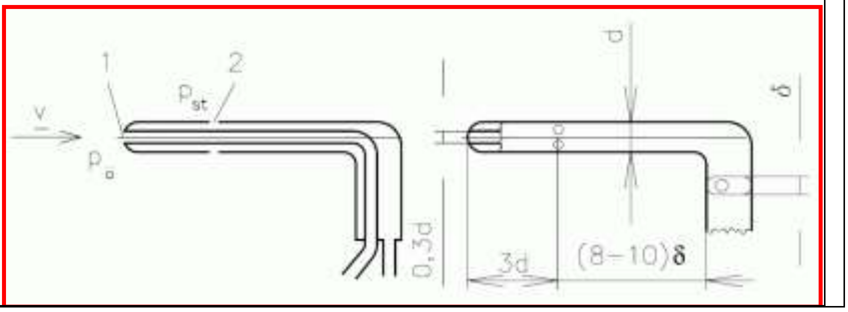


8. Describe the measurement of velocity using a Pitot-static probe (Prandtl probe). Illustrate your explanation with a sketch! What additional capability does the Pitot-static probe (Prandtl probe) have over the Pitot probe?

The Pitot-static probe (Prandtl probe) consists of **two concentric pipes**. The **internal (Pitot) pipe** can measure the **total pressure at the stagnation point**. There are **static pressure taps** located at a specified distance from the nose of the Prandtl tube on **the outer pipe**, from where the outer pipe conveys the static pressure to the measurement terminal. By connecting the terminals at the other ends of the pipes to the terminals of a manometer, we can measure the total pressure, the static pressure, or the difference between them, the **dynamic pressure**. The **flow velocity** can be calculated from the dynamic pressure if the **density** of the flowing medium is known. The advantage of the Pitot-static probe (Prandtl probe) over the Pitot probe is that it can also be used in curved streamlines, as it measures the local static pressure.

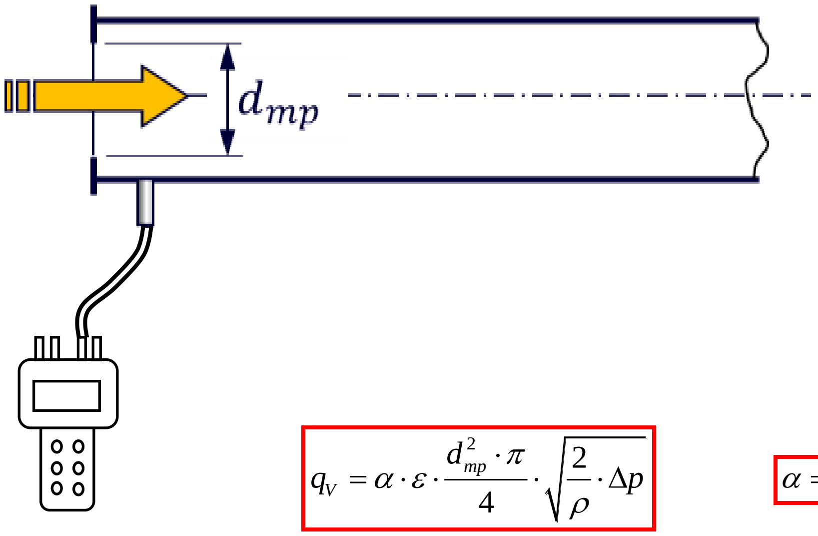
$$p_{dyn} = \Delta p = p_{\ddot{o}} - p_{st};$$

$$v = \sqrt{\frac{2 \cdot p_{dyn}}{\rho}} = \sqrt{\frac{2 \cdot \Delta p}{\rho}}$$



9. Describe a volume flow rate measurement that contains an inlet orifice. Provide a sketch that includes the inlet orifice, the locations of the pressure taps, and the connections to a manometer. Indicate which ports of the manometer measure the higher and lower pressure values. Provide the equation used to determine the volume flow rate with an inlet orifice and specify the names and units of the quantities included. In your description, provide values for the contraction ratio and the expansion number!

A **plate** with a **sharp-edged hole** in the middle is connected to the inlet end of a pipeline in an **airtight manner**. The diameter of the hole on the plate is smaller than the inner diameter of the pipe. A **static pressure tap** is created directly downstream of the plate. During the volume flow rate measurement, the pressure that can be measured downstream of the inlet orifice is compared to **atmospheric pressure** (Δp)! Since the inlet orifice sucks air from the surrounding space, a pressure lower than atmospheric pressure should be expected on the static pressure tap!



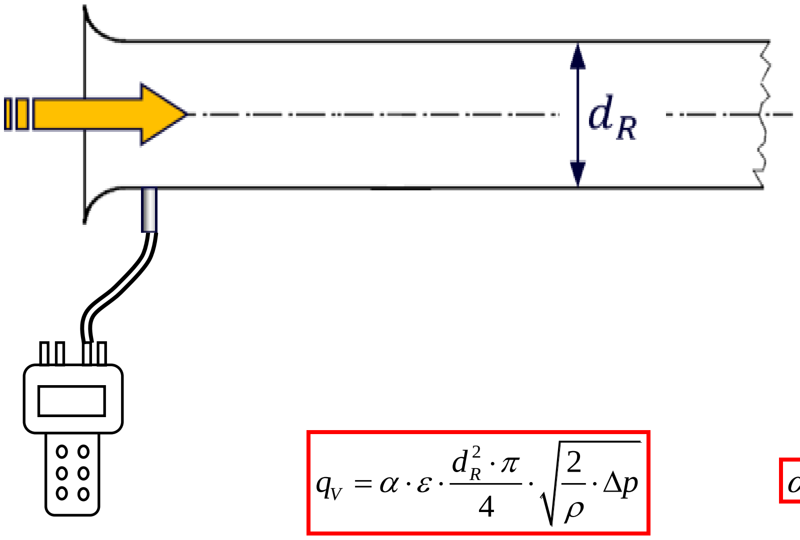
$$\boxed{q_V = \alpha \cdot \varepsilon \cdot \frac{d_{mp}^2 \cdot \pi}{4} \cdot \sqrt{\frac{2}{\rho} \cdot \Delta p}} \qquad \boxed{\alpha = 0,6}$$

q_V: volume flow rate, [m^3/s]
α: contraction ratio, [-]
ε: expansion number, [-]
d_{mp}: diameter of the tightest cross-section,[m]
Δp: pressure difference, [Pa]
ρ density of the fluid medium, [kg / m^3]

The expansion number can be taken as 1 as long as the pressure change is below 5000 Pa.

10. Describe a volume flow rate measurement that contains a bell mouth inlet. Provide a sketch that includes the bell mouth inlet, the locations of the pressure taps, and the connections to a manometer. Indicate which ports of the manometer measure the higher and lower pressure values. Provide the equation used to determine the volume flow rate with a bell mouth inlet and specify the names and units of the quantities included. In your description, provide values for the contraction ratio and the expansion number!

At the inlet end of a pipeline a bell mouth inlet is installed, which is a **rounded, streamlined** element, which typically causes **minimal** flow disturbance. Downstream of the bell mouth inlet static pressure taps are created on the pipe-section with a **constant diameter**. During volume flow measurements, the pressure of **the static pressure tap** is compared to the atmospheric pressure (Δp)! Since the intake element sucks air from the surrounding space, a pressure lower than atmospheric pressure should be expected on the static pressure tap!



$$\boxed{q_V = \alpha \cdot \varepsilon \cdot \frac{d_R^2 \cdot \pi}{4} \cdot \sqrt{\frac{2}{\rho} \cdot \Delta p}}$$

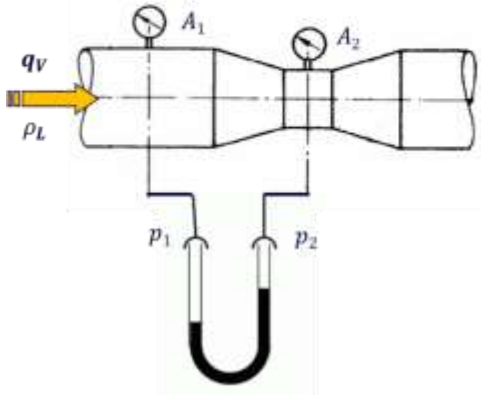
$$\boxed{\alpha \approx 1}$$

q_V: volume flow rate, [m^3/s]
α: contraction ratio, [-]
ε: expansion number, [-]
d_R: inner diameter of the duct, [m]
Δp: pressure difference, [Pa]
ρ density of the fluid medium, [kg / m^3]

The expansion number can be taken as 1 as long as the pressure change is below 5000 Pa.

11. Describe a volume flow rate measurement that contains a Venturi flowmeter. Provide a sketch that includes the Venturi flowmeter, the locations of the pressure taps, and the connections to a manometer. Indicate which ports of the manometer measure the higher and lower pressure values. Provide the equation used to determine the volume flow rate with a Venturi flowmeter and specify the names and units of the quantities included.

A Venturi flowmeter consists of a convergent nozzle and a divergent nozzle, which accelerates and then decelerates the flow of the medium. As a result of the acceleration, according to Bernoulli's equation, the pressure of the medium decreases, the extent of which can be expressed using Bernoulli's equation. The pressure drop in the convergent nozzle is measured during the volume flow rate measurement since typically no significant flow loss occurs here.



veszteségmentes, (leválás mentes), áramlás kialakítására törekszünk

Continuity:

$$q_V = v_1 \cdot \frac{D_1^2 \cdot \pi}{4} = v_2 \cdot \frac{D_2^2 \cdot \pi}{4} \quad \text{if } \rho\text{=const.}$$

Bernoulli's equation

$$p_1 + \frac{\rho}{2} \cdot v_1^2 + \rho \cdot U_1 = p_2 + \frac{\rho}{2} \cdot v_2^2 + \rho \cdot U_2 \quad \text{if the flow is lossless}$$

From which the volume flow rate can be determined:

$$q_V = \frac{D_1^2 \cdot \pi}{4} \cdot \sqrt{\frac{\Delta p}{\frac{\rho}{2} \cdot \left[\left(\frac{D_1}{D_2}\right)^4 - 1\right]}} = A_1 \cdot v_1$$

q_V: volume flow rate, $[q_V] = m^3/s$
$D_{1,2}$: 1: pipe diameter, 2: tightest cross-section [m]
$v_{1,2}$: 1: velocity in the pipe, 2: velocity in the tightest cross-section [m/s]
Δp: pressure difference measured with the manometer [Pa]
ρ: density of the fluid medium [kg / m^3]

12. Compare volume flow rate measurement methods based on velocity measurements to flow contraction methods! List their advantages and disadvantages!

ASPECT	**CONTRACTION**	**VELOCITY-BASED**
1/ Intrusiveness	**“ – ”** Considerable losses ⇒ the state of operation might change ⇔ needs to be considered for inclusion during the design stage	**“ + ”** Negligible intrusiveness (taps in the walls)
2/ Monitoring of operating conditions that change over time	**“ + ”** Continuously tracks the unsteady flow rate	**“ – ”** Does not track the flow rate continuously (integration over the surface) (⇔ correction..?)
3/ Requirements	**“ – ”** Strict (manufacturing, installation, system is the stopped …)	**“ + ”** Moderate (no requirements, only recommendations; continuous operation of the system …)
4/ Expenses	**“ – ”** High (manufacturing, installation, operation: losses need to be covered)	**“ + ”** Moderate
5/ Accuracy	**“ + ”** High (limited uncertainty, guaranteed by the standard) Legally defendable!	**“ – ”** Moderate (degree of uncertainty is not guaranteed) It can be legally questioned!

13. Define absolute and relative errors (uncertainties)! How can one determine the relative error of a quantity calculated from several measured values?

In engineering practice, the measured quantities are only known with a measurement uncertainty. To determine how reliable the results are, error calculations must be carried out. Let X denote the measured quantity, and δX the measurement error of that quantity. The correct way of providing measurement results is the following:

$$\boxed{X \pm \delta X}$$

where δX is the absolute error of quantity X,

$$\boxed{\frac{\delta X}{X}}$$

while this ratio is the relative error (usually given in a percentage form).

In most cases, the measurement error is caused by the inaccurate reading of the measurement device. The reading error roughly corresponds to the scale division of the given instrument, e.g., with a manometer, the deflection of the measurement fluid is read on a [mm] scale of the measurement instrument. Here, the reading error for the liquid column deflection is 1 mm. The measurement error of a quantity calculated from several independently measured values is the sum of the error of the measured quantities. The absolute error can be calculated by the following equation:

$$\delta R = \sqrt{\sum_{i=1}^{n} \left(\delta X_i \cdot \frac{\partial R}{\partial X_i} \right)^2}$$

where:

R	the calculated quantity,
X_i	*i*-th element of the *n* measured quantities
δX_i	absolute error of measuring X_i
δR	absolute error of the calculated quantity

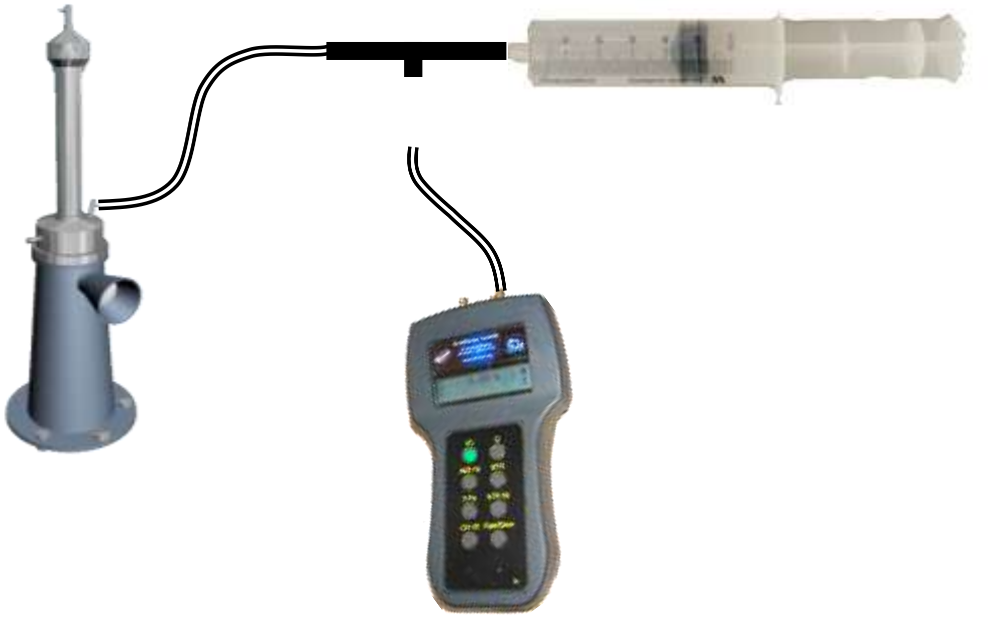
14. Describe the purpose, process, and result of a calibration. Show an example for the application of a calibration.

A task that often occurs in engineering practice is calibrating a less accurate, non-standard device with a more accurate, standardized device to make the less accurate, non-standard device suitable for measurements. During calibration, a **physical quantity can be measured in several states**, but it is important to carry out the measurements on the two devices **simultaneously**. Next, a **relationship** that converts the value measured with the less accurate device to the value measured with the more accurate device (**calibration equation**) is determined.

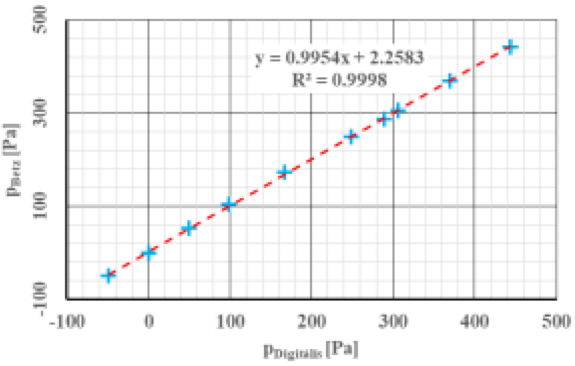
Example: Calibration of the EMB-001 digital pressure gauge using a Betz manometer

The **overpressure** branch of the **Betz manometer** and the **digital manometer** is connected to a syringe using a **T junction** and **silicone tubing**. By pushing in and pulling out the **syringe**, **overpressure/depression** (pressure below atmospheric pressure) is produced, which is simultaneously measured at several points with the **Betz manometer** and the **digital manometer**. By plotting the pressure measured with a digital manometer (x axis) as a function of the pressure values measured on the Betz manometer (z axis), we get the **calibration diagram**. By fitting a **regression line (linear trend line)** to the measurement points, the equation can be determined, which can be used to determine the pressures measured with a more accurate device (Betz manometer) from the pressure values measured on a digital manometer (a less precise measurement).

Layout:



Results


y = 0.9954x + 2.2583
R² = 0.9998
p_Betz [Pa]
500
300
100
-100
-100
0
100
200
300
400
500
p_Digitalis [Pa]