Flow Measurements
BMEGEÁTMW03

"laser-optical flow measurements"

Handout
by

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Contents, main topics of the lecture:

4. Characterisation of particle-laden mixtures

Aerosols

<u>Definition of aerosols:</u>

Aerosols are defined as being gas-particle mixtures in quasi-stable state. The mixture contains partly gas as primary (or carrier) phase and partly solid or liquid particulate matter (as secondary phase).

The quasi-stable state means that the characteristics of the mixture in a given volume (e.g. particle number, mass concentration) do not change significantly, i.e. "nearly stable" in time.

Changing of the characteristics of the mixture can occur due to the:

- settling out of larger particles from the given volume of the mixture, or
- diffusion and agglomeration of the small particles.

Both may cause increase or decrease of mass of the particles in the fixed volume, hence may cause changing of the characteristics of the mixture.

Diameter (x [μm]) range of the particles in aerosols: $0{,}01\mu m \le x \le 50\ \mu m$

Note, that the lower & upper limiting values are not strict limiting values: "0,01μm" & "50μm" means that approx. a few hundreds & few times ten microns can be considered as the limits of the diameter range of aerosols.

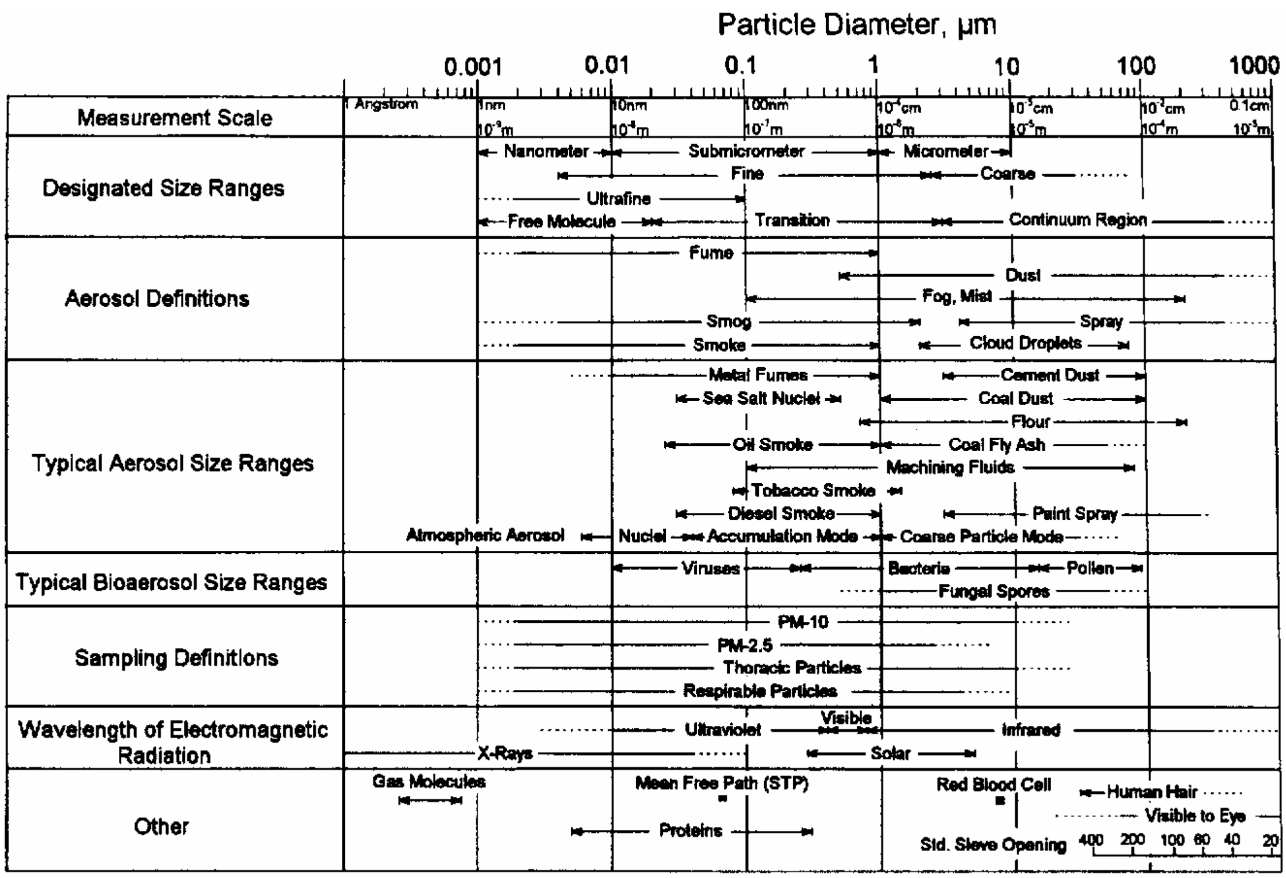
> ***Note*:**
> $1\mu m = 10^{-3}\ mm = 10^{-6}\ m$
> *The resolution of sensitivity of a human finger tip is about 40 microns.*
> *The human hair's diameter is 40÷100 microns.*
> *The average height of surface roughness of a bearing ball is aprrox. 0,01 micron.*
> *The diameter of seeding particles for Laser Doppler Velocimetry and flow visualization (spherical oil smoke droplets) is approx. 1÷3 microns.*

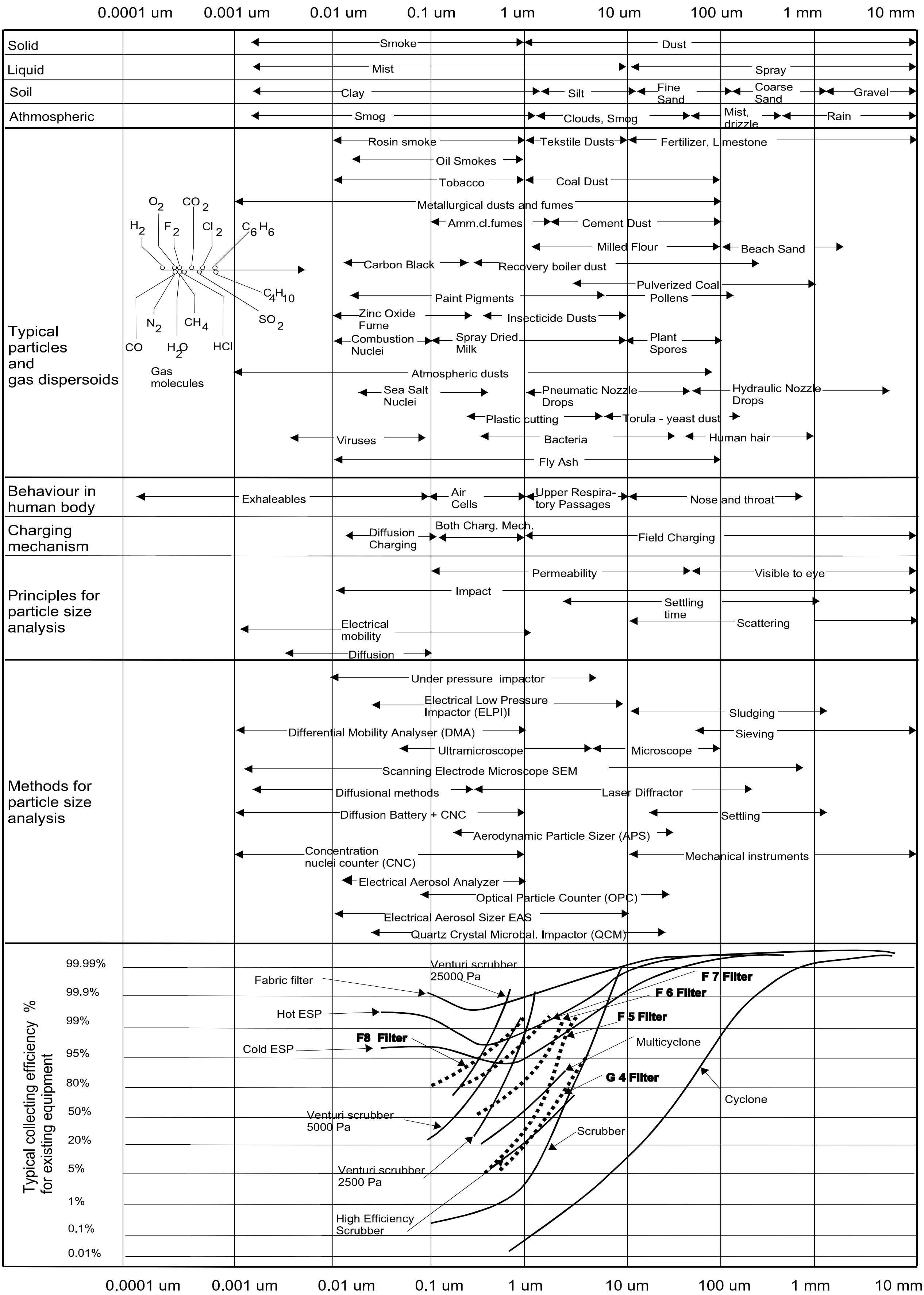
Types and sizes of particles

Dust:
size range: $x \geq 0{,}2$ [µm]
description: solid particles, produced by breaking or attrition, abrasion, wearing of solid substances, perceptible to the eye, the diameter is larger than the wave length of light.

Smoke (fume):
size range: $x \leq 1$ [µm]
description: solid or liquid particles or droplets, originated from condensation or chemical reaction, in most cases chain-like structures. Produced at combustion, chemical processes etc.

Mist (fog):
size range: $0{,}1 \leq x \leq 200$ [µm]
description: liquid droplets originated from steam condensation or by atomization, spraying. The mist droplets and the saturated steam of that liquid are in equilibrium state.

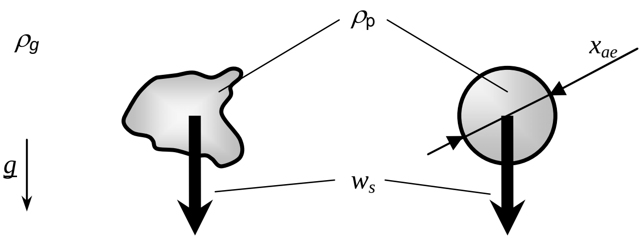



0.0001 um
0.001 um
0.01 um
0.1 um
1 um
10 um
100 um
1 mm
10 mm
Solid
Smoke
Dust
Liquid
Mist
Spray
Soil
Clay
Silt
Fine Sand
Coarse Sand
Gravel
Athmospheric
Smog
Clouds, Smog
Mist, drizzle
Rain
Typical particles and gas dispersoids
Rosin smoke
Tekstile Dusts
Fertilizer, Limestone
Oil Smokes
Tobacco
Coal Dust
Metallurgical dusts and fumes
Amm.cl.fumes
Cement Dust
Milled Flour
Beach Sand
Carbon Black
Recovery boiler dust
Pulverized Coal
Paint Pigments
Pollens
Zinc Oxide Fume
Insecticide Dusts
Combustion Nuclei
Spray Dried Milk
Plant Spores
Atmospheric dusts
Sea Salt Nuclei
Pneumatic Nozzle Drops
Hydraulic Nozzle Drops
Plastic cutting
Torula - yeast dust
Viruses
Bacteria
Human hair
Fly Ash
O2
CO2
H2
F2
Cl2
C6H6
C4H10
N2
CH4
SO2
CO
H2O
HCl
Gas molecules
Behaviour in human body
Exhaleables
Air Cells
Upper Respiratory Passages
Nose and throat
Charging mechanism
Diffusion Charging
Both Charg. Mech.
Field Charging
Principles for particle size analysis
Permeability
Visible to eye
Impact
Settling time
Electrical mobility
Scattering
Diffusion
Methods for particle size analysis
Under pressure impactor
Electrical Low Pressure Impactor (ELPI)I
Sludging
Differential Mobility Analyser (DMA)
Sieving
Ultramicroscope
Microscope
Scanning Electrode Microscope SEM
Diffusional methods
Laser Diffractor
Diffusion Battery + CNC
Settling
Aerodynamic Particle Sizer (APS)
Concentration nuclei counter (CNC)
Mechanical instruments
Electrical Aerosol Analyzer
Optical Particle Counter (OPC)
Electrical Aerosol Sizer EAS
Quartz Crystal Microbal. Impactor (QCM)
Typical collecting efficiency % for existing equipment
99.99%
99.9%
99%
95%
80%
50%
20%
5%
1%
0.1%
0.01%
Fabric filter
Venturi scrubber 25000 Pa
F 7 Filter
F 6 Filter
Hot ESP
F 5 Filter
F8 Filter
Cold ESP
Multicyclone
G 4 Filter
Cyclone
Venturi scrubber 5000 Pa
Scrubber
Venturi scrubber 2500 Pa
High Efficiency Scrubber
0.0001 um
0.001 um
0.01 um
0.1 um
1 um
10 um
100 um
1 mm
10 mm

Size of particles:
In case of spherical particles the diameter is denoted by x. Elsewhere usually it is denoted by d_p.

How to define the size of non-spherical particles?
It is needed to introduce the equivalent diameter.
Various types of equivalent diameter can be defined based on
- geometrical,
- **aerodynamic** and
- optical equivalence.

For particle dynamics the most relevant is to know the **x_{ae} aerodynamic equivalent diameter**, that is defined to be the diameter of a spherical particle from the same material (ρ_p=same) as the real particle, settling with the same w_s settling velocity in the same gas (ρ_g=same).



Aerodynamic equivalent diameter, x_{ae}

Average relative distance (a/x) between neighboring particles in gas:

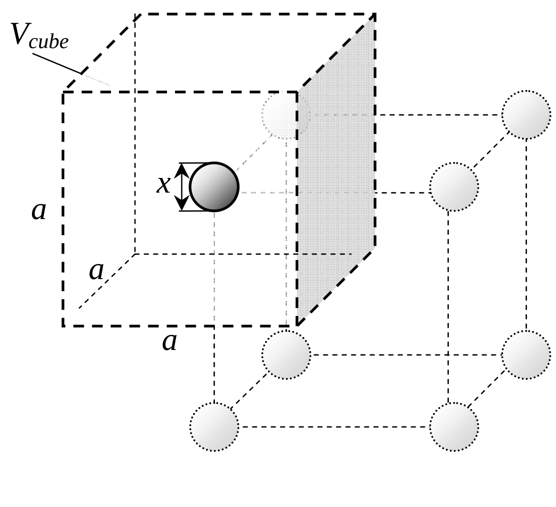
Let's calculate the c [kg/m^3] mass concentration of n particles evenly distributed in a particle-gas mixture having a volume of V_{g+p}. Let's assume that each particle is sitting in the center of a cube. (see Figure below).
The concentration can be calculated:

$$c = \frac{\sum m_p}{V_{g+p}} \cong \frac{\sum m_p}{V_{g+p}} = \frac{\sum V_p \cdot \rho_p}{V_{g+p}} = \frac{n \cdot \frac{x^3 \cdot \pi}{6} \cdot \rho_p}{n \cdot a^3} = \frac{\frac{x^3 \cdot \pi}{6} \cdot \rho_p}{a^3}$$

where c [kg/m^3] mass concentration, a [m] average distance between particles, ρ_p [kg/m^3] density of particles, n is number of particles.

For the average relative distance (a/x) between neighboring particles in gas we get:

$$\frac{a}{x} = \sqrt[3]{\frac{\rho_p \cdot \pi}{6 \cdot c}},$$



Example:
If we consider monodisperse particle distribution where all particles have x=3μm diameter with ρ_p=2000kg/m^3, and c = 10 g/m^3 (that is relatively very high concentration of particles), the a/x = 47.
a) It means that aerosols are very dilute mixtures: the neighboring particles are far from each other (approx. a=5ms when x=10cm would be the particle diameter). Hence, the possibility of collision, momentum exchange between two particles is relatively small.
b) In 1cm^3 volume there are 350000 particles for c=10 g/m^3. Even in case of small 0,1 g/m^3 concentration we get 3500 particle in 1cm^3 volume. Notwithstanding that it is a dilute mixture the number of particles is very high in given volume even for low concentration. That is important to know when very strict demand is defined on air quality (e.g. at surgery rooms, when dealing with toxic or infective particles is concerned)

c[g/m^3]	$\frac{a}{x}$	N [db/cm^3]
10	47	350.000
1	101	35.000
0.1	218	3.500

Conclusion:
- in case of usual particle concentration values the particle-laden flows are very dilute mixtures. (the distance between neighboring particles is very large).
- particles are present with very high number even in particle-gas mixtures having very low concentration.

Characterization of particle assembly:

Particle size distribution curves:
Considering polydisperse particle distribution with size range of $x_{min} < x < x_{max}$
Cumulative or undersize distribution related to number of particles: $Q_0=N/N_{tot}=f(x)$. Q_1, Q_2, Q_3.
Subscript denotes: 0: related to number of…
1: related to length of... (1D – one dimensional quantity)
2: related to surface of…(2D – two dimensional quantity)
3: related to volume or mass of…(3D – three dimensional quantity)
If $Q_0=N/N_{tot}=f(x)$ and the overall number of particles N_{tot} are known, the number of particles in the size range between [x] and [x+Δx] can be calculated:

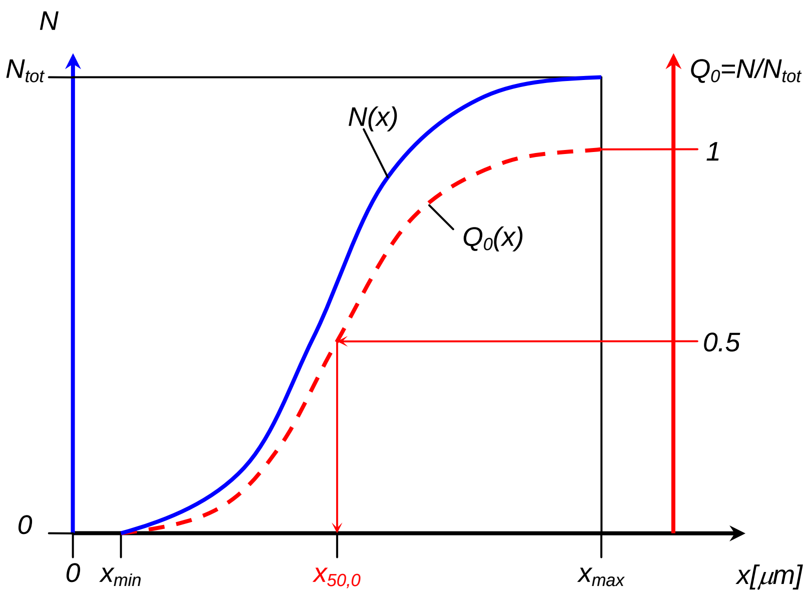
$$\Delta N = N_{tot} \cdot \frac{dQ_0}{dx} \cdot \Delta x\ .$$

Cumulative or undersize distribution related to number of particles: Q_0. The value of $Q_0(x)$ for given x gives information how many percentage of N_{tot} particles have smaller diameter that x.
Taking the tangent of the Q_0 curve (see Figure below) is denoted by:

$$q_0=\frac{dQ_0}{dx}\ ,$$

we get for the number of particles in the range between [x] and [x+Δx]:

$$\Delta N = N_{tot} \cdot \frac{dQ_0}{dx} \cdot \Delta x = N_{tot} \cdot q_0 \cdot \Delta x$$

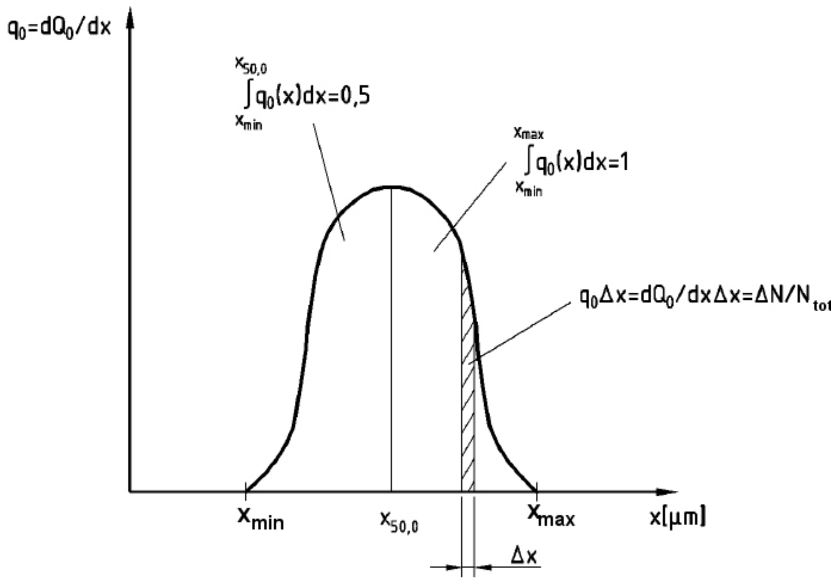


Relation/conversion between distributions related to various quantities:
For example conversion of Q_0 to Q_3:

$$Q_3(x) = \frac{\int_{x_{min}}^{x} x^3 \frac{\pi}{6} N_{tot} \frac{dQ_0}{dx} dx}{\int_{x_{min}}^{x_{max}} x^3 \frac{\pi}{6} N_{tot} \frac{dQ_0}{dx} dx} = \frac{\int_{x_{min}}^{x} x^3 q_0 dx}{\int_{x_{min}}^{x_{max}} x^3 q_0 dx}.$$

When q_0 cumulative distribution related to the particle number is given (see Figure below), we can obtain the average diameter of the particle distribution ($\bar{x}_0$) related to particle number:

$$\bar{x}_0 = \frac{1}{N_{tot}} \int_{x_{min}}^{x_{max}} x \cdot N_{tot} \cdot q_0 dx = \int_{x_{min}}^{x_{max}} x \cdot q_0 dx .$$



5. Particles in gas flow: particle dynamics

Effect of particles on the gas flow

Navier-Stokes equation extended with considering the influence of the particles' forces acting on the carrier gas phase:

$$\boxed{\frac{\partial \underline{v}}{\partial t} + \mathrm{grad}\frac{v^2}{2} - \underline{v} \text{ x } \mathrm{rot}\underline{v} = \underline{g} - \frac{1}{\rho_g}\mathrm{grad}p + \nu\Delta\underline{v} + \underline{t}}$$

where $\underline{t}\left[N / kg_{gas}\right]$ is the force acting to the gas from particles in 1 kg of gas:

$$\underline{t} = -\frac{n\underline{F}}{\rho_g}$$

$n[\text{piece/m}^3]$: particle number concentration
$\underline{F}\ [\text{N / piece}]$: aerodynamic force acting on one particle
$\rho\ [\text{kg / m}^3]$: gas density

The effect of particle phase on the flow field can be neglected, if $\frac{c}{\rho_g} \langle\langle 1$ and the particle acceleration $\frac{d\underline{u}_p}{dt}$ is in the same order of magnitude as the carrier gas-phase acceleration $\frac{d\underline{v}}{dt}$.

Therefore in case of $\frac{c}{\rho_g}\frac{d\underline{u}_p}{dt} \langle\langle \frac{d\underline{v}}{dt}$, the effect of particle phase on the flow field can be neglected. From other viewpoint we can say that in this dilute mixture one single particle moving in the gas cannot change the gas' momentum, but one particle's movement is influenced by the carrier gas, see in next chapter: defining the aerodynamic force acting on the particles.

Aerodynamic (drag) force acting on a single particle:

Particle Reynolds number: $\mathrm{Re}_p = \frac{w \cdot x}{\nu}$ is small, viscosity is dominant in a fluid flow around the particle (using the relative co-ordinate system fixed to the particle)

Stokes:
If $\mathrm{Re}_p < 0.1$, the well known Stokes relation for $\underline{F}_d$ drag force acting on a particle (having diameter x) moving with $\underline{w}$ relative velocity in the gas (μ is dynamic viscosity) is:

$$\boxed{\underline{F}_d = \underline{F}_{Stokes} = 3 \cdot \pi \cdot \mu \cdot x \cdot \underline{w}}$$

where $\underline{v} = \underline{u} + \underline{w}$
$\underline{v}$ absolute (gas) velocity
$\underline{u}$ particle velocity
$\underline{w}$ relative velocity

Drag coefficient of the particle?

$$c_d = \frac{\left|\underline{F}_d\right|}{\frac{\rho}{2} w^2 \cdot A_{ref}} = \frac{\left|\underline{F}_d\right|}{\frac{\rho}{2} w^2 \frac{x^2 \pi}{4}}$$

Substituting the Stokes relation for $\underline{F}_d$ into equation of c_d, we get a very simple form for the drag coefficient of a sphere:

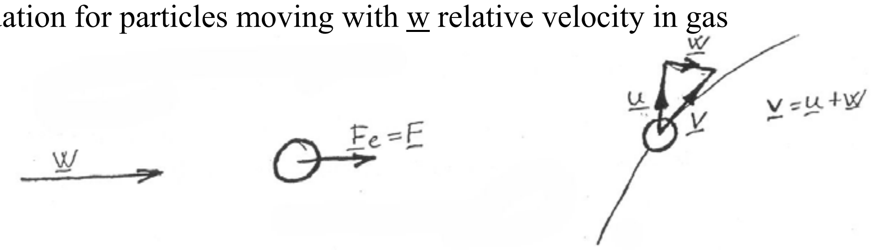
$$\boxed{c_d = \frac{24}{Re_p}}.$$

The above form can be used only in cases when the particle Reynolds-number is lower than 0.01. Researchers in this field obtained various forms for particle Reynolds-number corrected Stokes' drag coefficient based on further experiments or nowadays morely based on numerical simulations. For example:

a) the Oseen's relation: $c_e = \frac{24}{Re_p}\left(1 + \frac{3}{16} Re_p\right)$, that is valid when $Re_p < 5$.

b) the Michaelides's relation: $c_d = \frac{24}{Re_p} \cdot \left(1 + 0{,}15 \cdot Re_p^{0,687}\right)$ is valid when $0.1 < Re_p < 1000$.

Momentum equation for particles moving with $\underline{w}$ relative velocity in gas



Due to Newton's 2nd law the particle's momentum equals to the sum of the acting forces. Forces: gravity force and drag force (Usually we may neglect the bouyancy force).

$$m_p \frac{d\underline{u}}{dt} = \underline{F}_g + \underline{F}_d$$

$$\frac{x^3 \pi}{6} \rho_p \frac{d\underline{u}}{dt} = \frac{x^3 \pi}{6} \rho_p \underline{g} + 3\pi\, \mu\, x\, \underline{w}$$

Dimensionless equation of motion of the particle:

As a usual non-dimensionalizing formulation procedure let's multiply the equation with $\frac{l_0}{v_0^2}$, where l_0 is a characteristic length (e.g. l_0=x), and v_0 is a characteristic velocity (e.g. v_0= average gas flow velocity)

$$\frac{x^3 \pi}{6} \rho_p \frac{d\underline{u}}{dt} = \frac{x^3 \pi}{6} \rho_p \underline{g} + 3\pi\, \mu\, x\, \underline{w} \qquad \Bigg|\cdot \frac{l_0}{v_0^2}\ .$$

Then we get the form of equation of motion:

$$\frac{d\frac{\underline{u}}{v_0}}{d\frac{t}{l_0/v_0}} = \frac{\underline{g}\, l_0}{v_0^2} + \frac{18\,\mu}{x^2 \rho_p} \frac{l_0}{v_0} \frac{\underline{w}}{v_0}\ .$$

Dimensionless momentum equation for particles (' denotes dimensionless quantities, e.g. $\underline{u}' = \frac{\underline{u}}{v_0}$).

$$\frac{du'}{dt'} = \frac{g\,l_0}{v_0^2} + \frac{18\,\mu}{x^2\rho_p}\frac{l_0}{v_0}\underline{w}'$$

Settling velocity of the particle (w_s):

Settling of particle of ρ_p density in a gas of ρ_g density:

$$\frac{x^3\pi}{6}\rho_p g = \frac{x^3\pi}{6}\rho_g g + 3\pi\,\mu\,x\,w_s$$

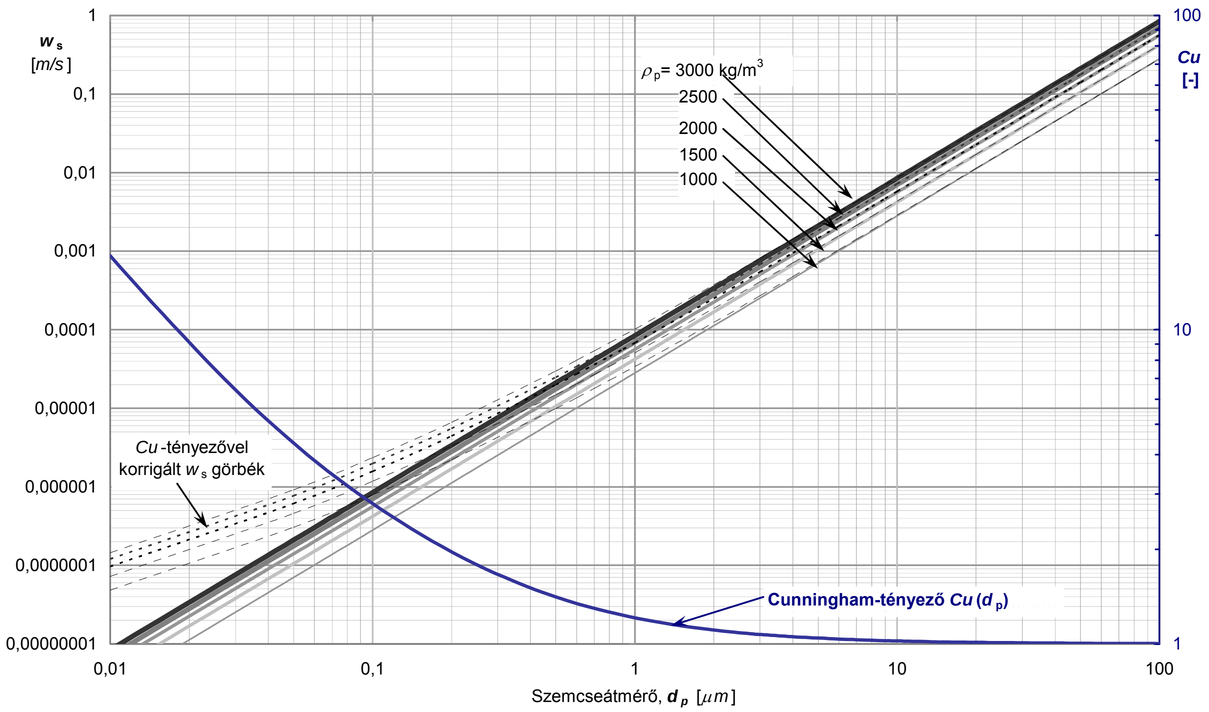
Settling velocity: $\boxed{w_s = \frac{x^2(\rho_p - \rho)g}{18\mu}}$ $\boxed{\text{if } \rho_p \rangle\rangle \rho \Rightarrow w_s = \frac{x^2\rho_p g}{18\mu}.}$

Correction of settling velocity due to the diffusion effect in submicron size-range:

$w_{s,corr} = Cu \cdot w_s$

where $Cu = 1 + \frac{2\,A\,\lambda}{x}$ is the <u>Cunningham coefficient (or Cunningham correction factor)</u>,

where A≈1.4, and λ is the mean free path of molecules, at room-temperature $\lambda = 6.5 * 10^{-2}$ μm).



w_s settling velocity (cont. lines) ad $w_{s,corr}$ corrected settling velocity (dashed lines)
Settling velocity as function of particle diameter and density

By neglecting the effect of the gravity field strength the dimensionless equation of motion of particles will turn to another form using the w_s settling velocity:

$$\frac{du'}{dt'} = \frac{l_0}{v_0^2}\underline{g} + \frac{18\,\mu}{x^2\rho_p}\frac{l_0}{v_0}\underline{w}' \cong \frac{18\,\mu}{x^2\rho_p}\frac{l_0}{v_0}\underline{w}' = \frac{g \cdot l_0}{w_s v_0}\underline{w}'$$

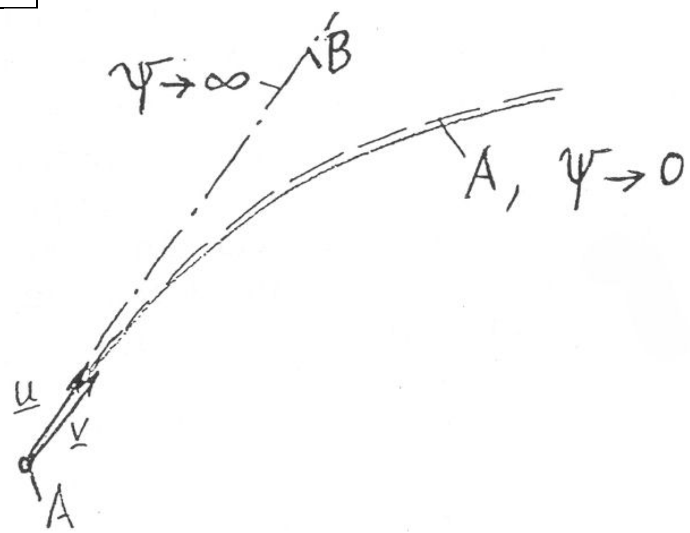
$$\boxed{\frac{du'}{dt'} = \frac{g \cdot l_0}{w_s v_0}\underline{w}'}$$

Introducing ψ inertia parameter will help us to evaluate the particle motion in gas flow:

$$\boxed{\psi = \frac{w_s v_0}{g \cdot l_0}}$$

Dimensionless momentum equation for particles with inertia parameter:

$$\boxed{\frac{d\underline{u}'}{dt'} = \frac{1}{\psi}\underline{w}' = \frac{1}{\psi}(\underline{v}' - \underline{u}')}$$



case A) dashed line in the upper figure

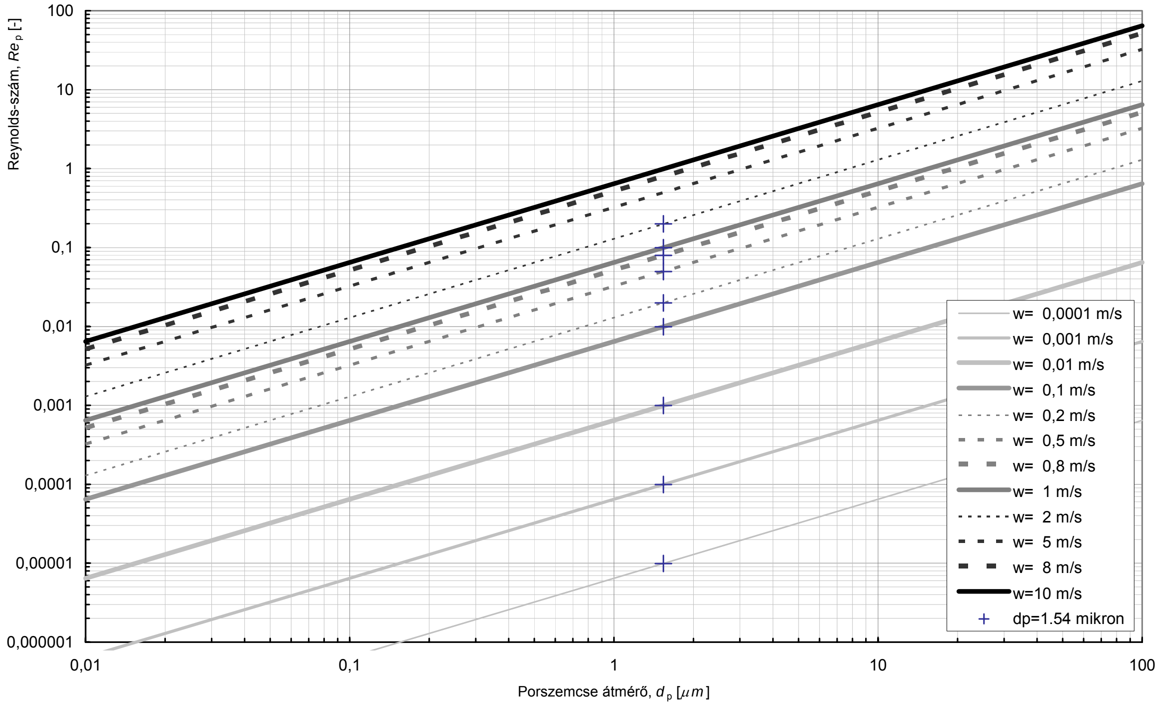
When $\psi \to 0$, for small (x) and/or light (ρ_p) particles, which settling velocity is small, or $w_s \to 0$, and if $(\underline{v}' - \underline{u}') \neq 0 \Rightarrow \frac{d\underline{u}'}{dt'} \to \infty$, hence particle move along the gas streamline, particle follow the carrier gas flow.

case B) dash-dot line in the upper figure

When $\psi \to \infty$, for large and/or heavy particles, which settling velocity is large, $\frac{1}{\psi} \to 0$, *consequently* $\frac{d\underline{u}'}{dt'} \to 0.$ hence particle move along its initial path, leaving the gas streamline.

Relative velocity of the particle: $\boldsymbol{w} = \mathbf{v} - \boldsymbol{u}$
Particle REYNOLDS-number (Re_p)

$$Re_p = \frac{w \cdot d_p}{\nu} = \frac{w \cdot d_p \cdot \rho_g}{\mu}$$



+ sign: data for particle generated by a professional SAFEX smoke generator for LDA measurements (d_p=1.5μm)

STOKES-formula for spherical particle ($Re_p < 0{,}25$) so-called STOKES-regime:

$$\boldsymbol{F}_e = 3\pi\,\mu\,d_p\boldsymbol{w}$$

$$c_e = \frac{24}{Re_p}$$

OSEEN 's formula to extend the validity above Stokes-regime:

$$c_e = \frac{24}{Re_p} \cdot \left(1 + \frac{3}{16} Re_p\right) \qquad Re_p < 5$$

$$c_e = \frac{24}{Re_p} \cdot \left(1 + \frac{Re_p^{\frac{2}{3}}}{6}\right) \qquad 3 < Re_p < 400$$

same by MICHAELIDES (1997):

$$c_e = \frac{24}{Re_p} \cdot \left(1 + 0{,}15 \cdot Re_p^{0{,}687}\right) \qquad 0{,}1 < Re_p < 1000$$

Charatceristic parameters of the primary & secondary phase:

- carrier fluid (primary phase)
- seeding /tracer particles (secondary phase)

Volume ratio:

$$\alpha_p = \frac{V_p}{V_g} = \frac{\frac{d_p^3 \pi}{6}}{a^3} = \frac{c_p}{\rho_p}$$

Mass loading ratio:

$$M = \frac{c_p}{\rho_g} = \alpha_p \frac{\rho_p}{\rho_g}$$

$$\frac{M}{\alpha_p} = \frac{\rho_p}{\rho_g}, \text{ or } \frac{\alpha_p}{M} = \frac{\rho_g}{\rho_p}$$

where:

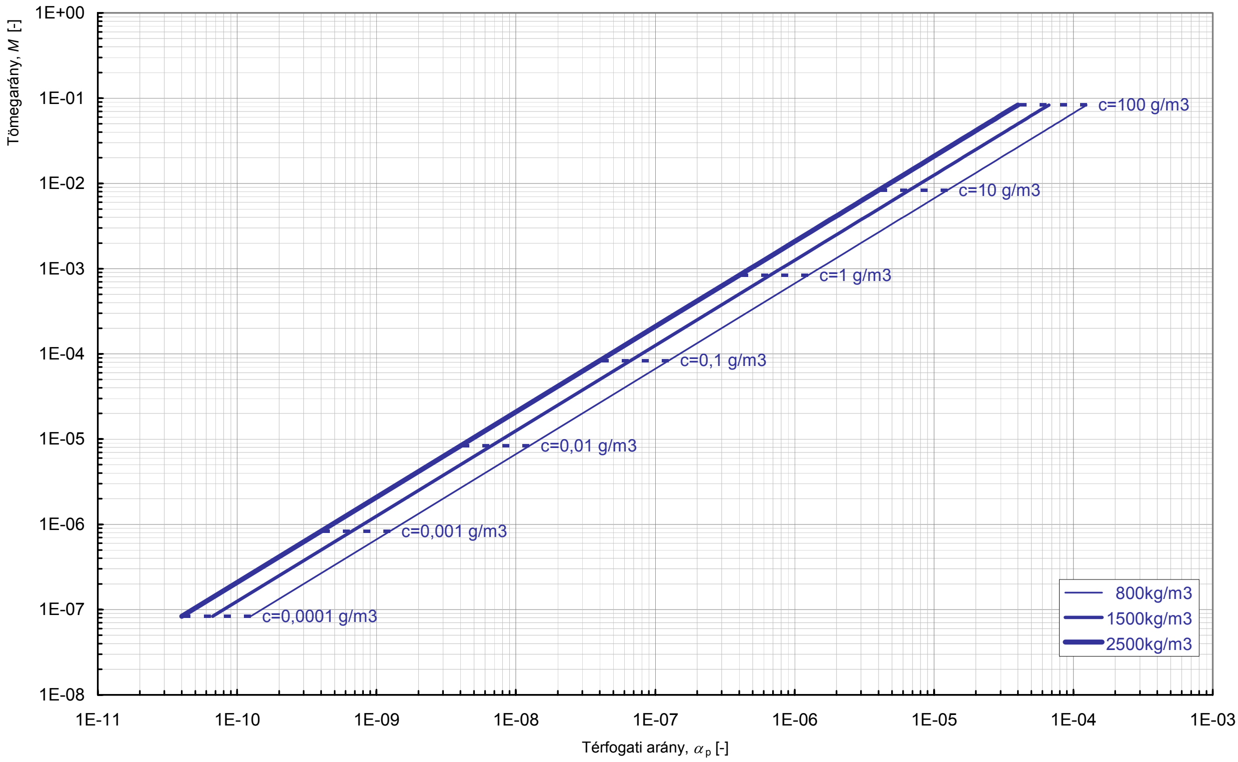
c_p: particle mass concentration

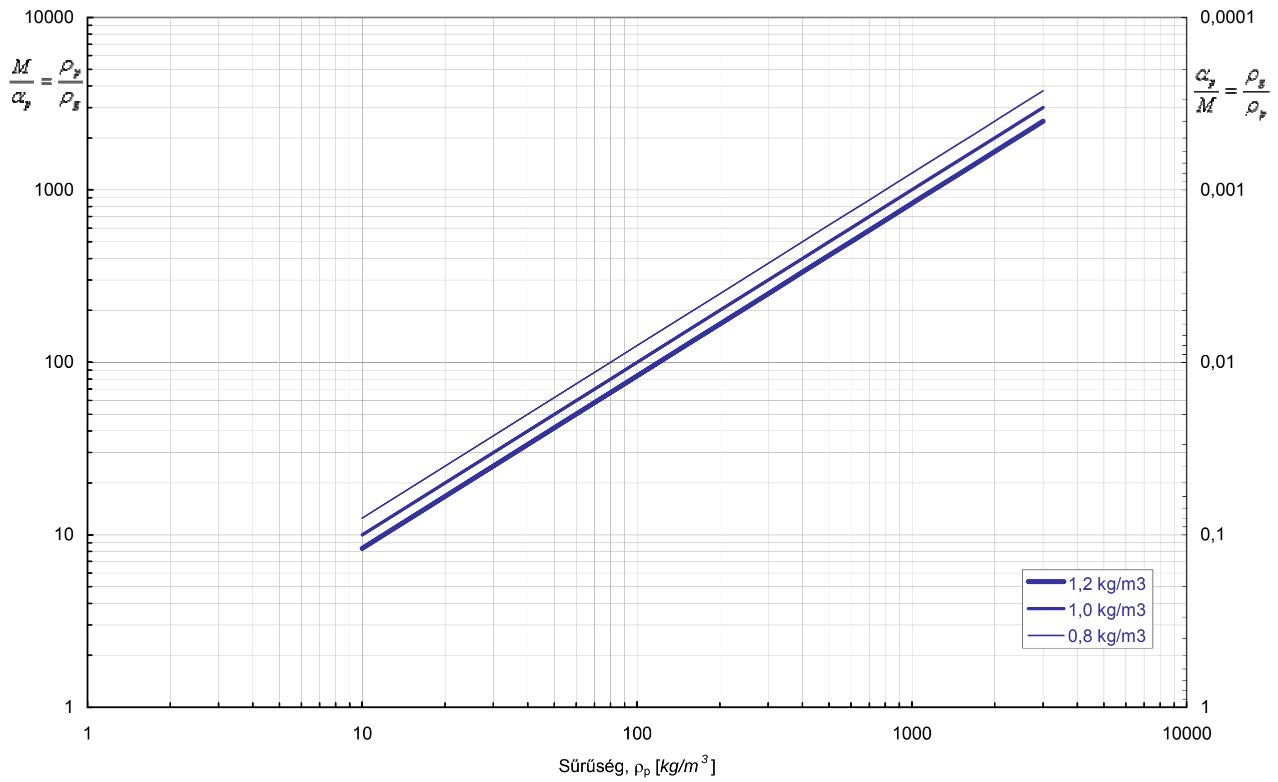
ρ_g: density of gas (carrier phase)

ρ_p: density of particle (material)

α_p		ρ_p [kg/m^3]		
		800	**1500**	**2500**
c_p [g/m^3]	**0,0001**	$1{,}3\cdot10^{-10}$	$6{,}7\cdot10^{-11}$	$4{,}0\cdot10^{-11}$
	0,001	$1{,}3\cdot10^{-9}$	$6{,}7\cdot10^{-10}$	$4{,}0\cdot10^{-10}$
	0,01	$1{,}3\cdot10^{-8}$	$6{,}7\cdot10^{-9}$	$4{,}0\cdot10^{-9}$
	0,1	$1{,}3\cdot10^{-7}$	$6{,}7\cdot10^{-8}$	$4{,}0\cdot10^{-8}$
	1	$1{,}3\cdot10^{-6}$	$6{,}7\cdot10^{-7}$	$4{,}0\cdot10^{-7}$
	10	$1{,}3\cdot10^{-5}$	$6{,}7\cdot10^{-6}$	$4{,}0\cdot10^{-6}$
	100	$1{,}3\cdot10^{-4}$	$6{,}7\cdot10^{-5}$	$4{,}0\cdot10^{-5}$

M		ρ_g[kg/m^3]		
		0,8	**1,0**	**1,2**
c_p [g/m^3]	**0,0001**	$1{,}3\cdot10^{-7}$	$1{,}0\cdot10^{-7}$	$8{,}3\cdot10^{-8}$
	0,001	$1{,}3\cdot10^{-6}$	$1{,}0\cdot10^{-6}$	$8{,}3\cdot10^{-7}$
	0,01	$1{,}3\cdot10^{-5}$	$1{,}0\cdot10^{-5}$	$8{,}3\cdot10^{-6}$
	0,1	$1{,}3\cdot10^{-4}$	$1{,}0\cdot10^{-4}$	$8{,}3\cdot10^{-5}$
	1	$1{,}3\cdot10^{-3}$	$1{,}0\cdot10^{-3}$	$8{,}3\cdot10^{-4}$
	10	$1{,}3\cdot10^{-2}$	$1{,}0\cdot10^{-2}$	$8{,}3\cdot10^{-3}$
	100	$1{,}3\cdot10^{-1}$	$1{,}0\cdot10^{-1}$	$8{,}3\cdot10^{-2}$





Table

a / d_p		ρ_p [kg/m^3]		
		800	**1500**	**2500**
c_p [g/m^3]	**0,0001**	1612	1988	2357
	0,001	748	923	1094
	0,01	347	428	508
	0,1	161	199	236
	1	75	92	109
	10	35	43	51
	100	16	20	24

N [db/mm^3]		ρ_p [kg/m^3]		
		800	**1500**	**2500**
c_p [g/m^3]	**0,0001**	0,07	0,04	0,02
	0,001	0,7	0,4	0,2
	0,01	7	4	2
	0,1	71	38	23
	1	707	377	226
	10	7074	3773	2264
	100	70736	37726	22635

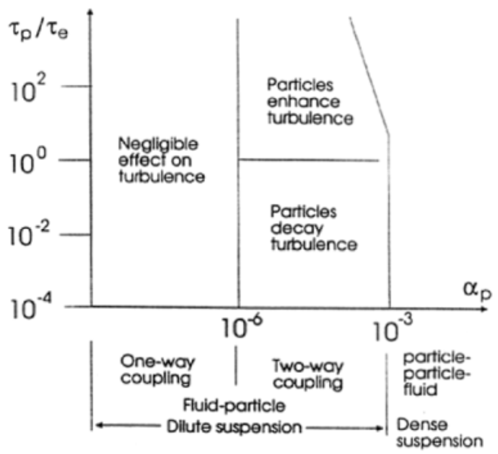
ELGHOBASHI (1994): „Turbulence modulation map": particle STOKES-number ($St_p = \tau_p / \tau_e$) in function of the α_p

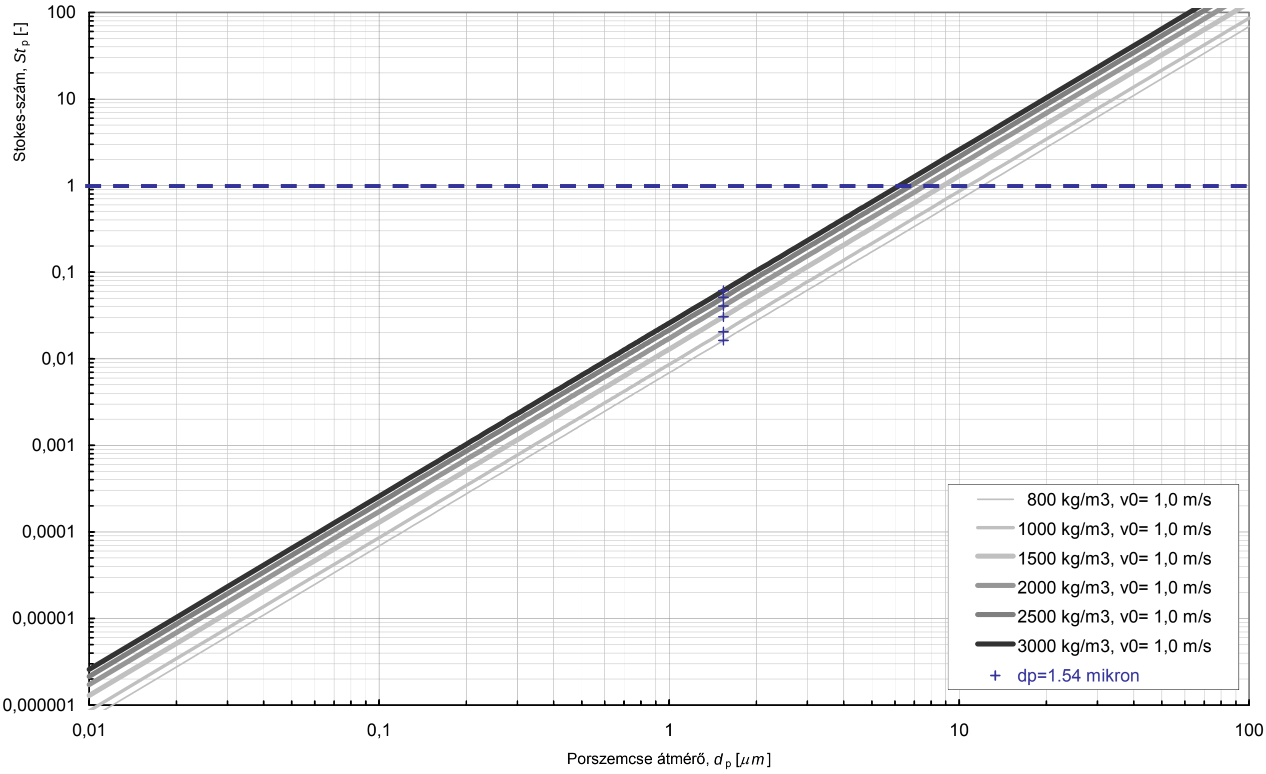
τ_p: characteristic (response) time of the particle

τ_e: characteristic time of the carrier fluid

dilute mixtures: $\alpha_p < 10^{-3}$

dense mixtures: $\alpha_p > 10^{-3}$





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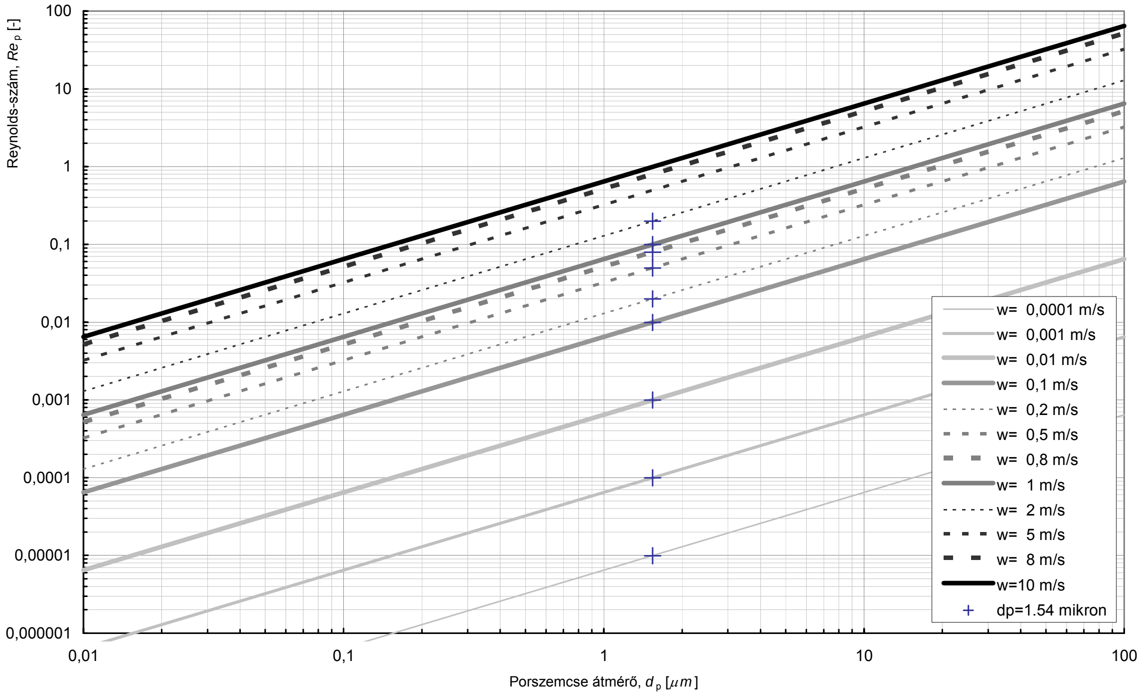

$$\boldsymbol{F}_{Stokes} = 3\pi\,\mu\,d_p\boldsymbol{w}\,, \qquad Re_p = \frac{w \cdot d_p}{\nu} = \frac{w \cdot d_p \cdot \rho_g}{\mu}\,, \qquad c_d = \frac{F_{Stokes}}{\frac{\rho_g}{2} w^2 \frac{d_p^2 \pi}{4}}\,, \quad c_d = \frac{24}{Re_p}$$

OSEEN $$c_d = \frac{24}{Re_p} \cdot \left(1 + \frac{3}{16} Re_p\right) \qquad Re_p < 5 \qquad (1)$$

OSEEN $$c_d = \frac{24}{Re_p} \cdot \left(1 + \frac{Re_p^{\frac{2}{3}}}{6}\right) \qquad 3 < Re_p < 400 \qquad (2)$$

MICHAELIDES $$c_d = \frac{24}{Re_p} \cdot \left(1 + 0{,}15 \cdot Re_p^{0{,}687}\right) \qquad 0{,}1 < Re_p < 1000 \qquad (3)$$

Settling velocity of the particle (and from when effect of buoyancy force is neglected):

$$w_s = \frac{(\rho_p - \rho_g) d_p^2 g}{18\mu} \cong \frac{\rho_p d_p^2 g}{18\mu}$$