

M2

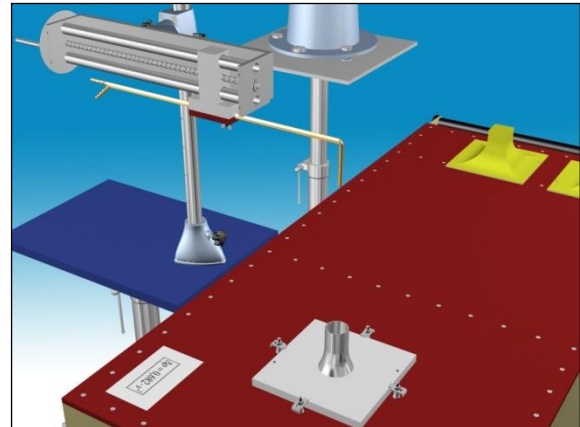
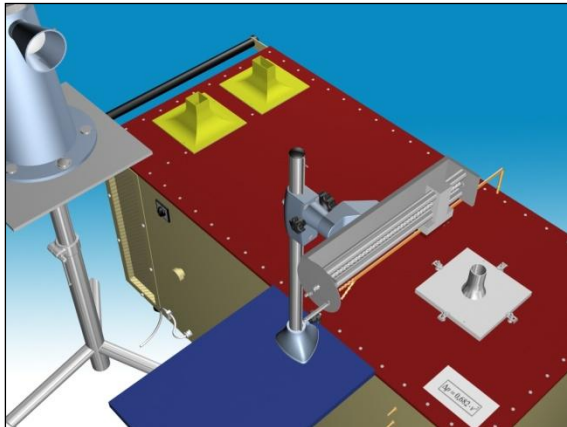
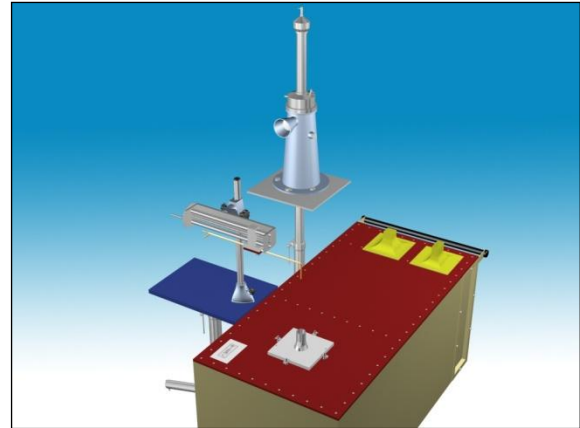
INVESTIGATION OF A FREE JET

1. Aim of the measurement

A free jet refers to a flow which is entering a stationary field through an orifice. The volume flow rate of the jet is negligible as compared to the surrounding space. Numerous applications for free jets are known. These include ventilation equipment, refrigeration and jet powered machinery. The objective of this measurement is to investigate the characteristics of a free jet.

2. Description of the measurement equipment

The air is delivered by a centrifugal fan, which is installed into the test rig. The shape of the free jet at the outlet is determined by the cross-section of the outlet nozzle (square, circular or rectangular). The homogenous velocity profile at the outlet is provided by the flow straighteners and the screens installed before the outlet section. By means of the two pressure taps placed on this section, the pressure difference can be measured, by which the outlet mean velocity can be calculated. The air velocity is controlled by a throttle valve, which can be found at the suction inlet of the test rig.



2.1 Summary of the characteristics of a free jet

A free jet entrains the air from its surroundings, and therefore the cross-section of the jet, the velocity profile and the volume flow rates changes. The objective of the measurement is to investigate these characteristics.

According to the experiments and the theoretical considerations, the structure of the free jet, with respect to the velocity profile, can be divided into two main parts. (see Figure 1.) Up to the so-called core distance (z_c), the axial velocity in the jet core fairly agrees with the outlet velocity. This zone is called to the initial section of the free jet. The core distance (z_c) by the scientific literature amounts to 5-6 d_0 . When $z > z_c$, the axial velocity in the core gradually decreases (i.e. $v_{\max} < v_0$), furthermore the measured velocity profiles in each cross-section are similar to each other. The edge of the jet is defined by the shear layer, where the velocity values rapidly drop to zero.

To investigate the further characteristics of a free jet, the momentum equation must be applied with the following simplifications:

- ❖ The jet flows into a motionless field
- ❖ The effect of the friction and field force (e.g. gravity) are neglected
- ❖ The outlet velocity (v_0) is constant in time
- ❖ The pressure deviation from the ambient pressure (p_0) owing to the air entrainment occurring in the shear layer, resulting in curved streamlines, is not taken into account.

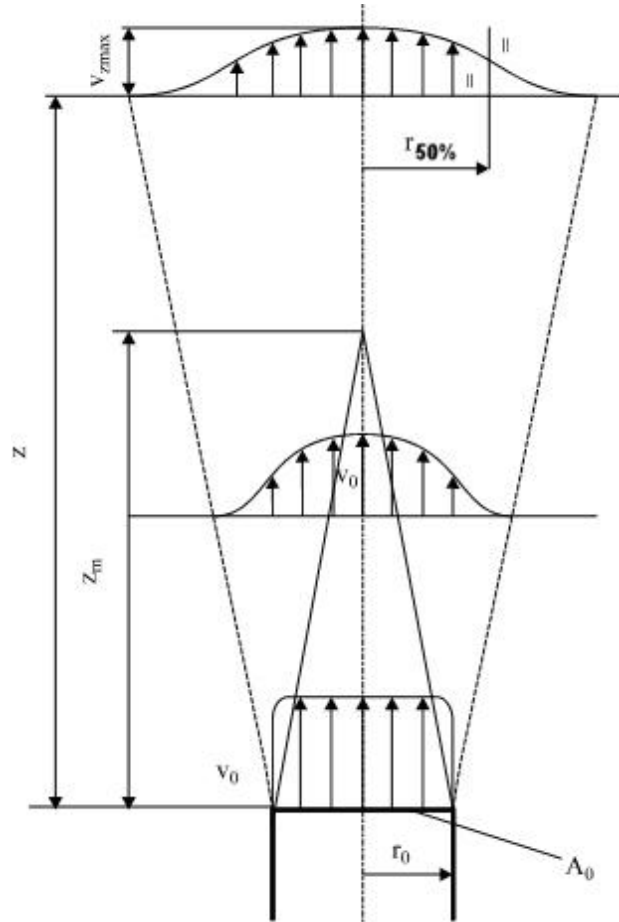


Figure 1: Velocity distribution in the free jet

The experiments have revealed that the negligence of the last effect (the pressure increase due to the curved streamlines) does not cause any considerable error (it is less than 1% of the outlet dynamic pressure). As a consequence of this, the force originating from the pressure acting on the control surface is neglected, i.e. $\int_A p dA \cong 0$, where A [m^2] denotes the surface of an optional area surrounding the jet (control surface). We therefore obtain the following approximation of the momentum equation: $\int_A v \rho v dA \cong 0$.

In any given cross-sections (A_s), which is perpendicular to the axis of the free jet, the momentum equation is equal to constant i.e. $\int_{A_s} v \rho v dA = C$, since on the sides of the control surface which are parallel to the axis it amounts to zero as a result of the symmetry, i.e. $\int v \rho v dA = 0$.

The following equation holds for all the surfaces which are perpendicular to the axis of the jet, such as the outlet cross-section:

$$I = \rho v_0^2 A_0 = \rho \int_{A_s} v^2 dA. \quad (1)$$

Let's make the normalized form of the preceding integral by using the following parameters:



In the case of a circular orifice:

- ❖ The maximum velocity in a certain velocity profile: v_{max}
- ❖ The radius where 50% of the maximum velocity value can be found (This can be determined from the velocity profile.): $r_{50\%}$

The normalized form of the integral:

$$I_{\phi} = v_{max}^2 \cdot r_{50\%}^2 \int \left(\frac{v}{v_{max}} \right)^2 \cdot 2\pi \cdot \left(\frac{r}{r_{50\%}} \right) \cdot d \left(\frac{r}{r_{50\%}} \right) \quad (2)$$

In the case of a rectangular (a x b) orifice:

- ❖ Maximum velocity in a certain velocity profile: v_{max}
- ❖ The x, y distance where 50% of the maximum velocity value can be found (This can be determined from the velocity profile.): $x_{50\%}, y_{50\%}$

The normalized form of the integral:

$$I_{\square} = v_{max}^2 y_{50\%} x_{50\%} \int \left(\frac{v}{v_{max}} \right)^2 \cdot d \left(\frac{x}{x_{50\%}} \right) \cdot d \left(\frac{y}{y_{50\%}} \right) \quad (3)$$

As the velocity profiles in the decelerating section ($z > z_c$) of the jet are similar, the normalized integral is constant for all the cases. Regarding equation (1), the axial velocity of the deceleration section can be expressed as follows:

In the case of a circular orifice:

$$\frac{v_{max}}{v_0} = \sqrt{\frac{r_0^2}{r_{50\%}^2} \cdot \frac{1}{\int \left(\frac{v}{v_{max}} \right)^2 \cdot \left(\frac{r}{r_{50\%}} \right) \cdot d \left(\frac{r}{r_{50\%}} \right)}} \quad (4)$$

In the case of a rectangular (a x b) orifice:

$$\frac{v_{max}}{v_0} = \sqrt{\left(\frac{a}{x_{50\%}} \right) \cdot \left(\frac{b}{y_{50\%}} \right) \cdot \frac{1}{\int \left(\frac{v}{v_{max}} \right)^2 \cdot d \left(\frac{x}{x_{50\%}} \right) \cdot d \left(\frac{y}{y_{50\%}} \right)}} \quad (5)$$

The values of $x_{50\%}$, $y_{50\%}$, $r_{50\%}$, which are representative of the development of the free jet, should be determined by measurements, as a function of the height (z).

In a certain 'z' height the normalized form of the $q = \int_{A_s} v dA$ volume flow rate is:

In the case of a circular orifice:

$$\left(\frac{q}{q_0} \right)_{\phi} = \frac{v_{max} \cdot r_{50\%}^2}{v_0 \cdot r_0^2} \int \frac{v}{v_{max}} \cdot \frac{r}{r_{50\%}} \cdot d \left(\frac{r}{r_{50\%}} \right) = \frac{v_{max} \cdot r_{50\%}^2}{v_0 \cdot r_0^2} \cdot T_{\phi} \quad (6)$$



In the case of a rectangular (a x b) orifice:

$$\left(\frac{q}{q_0}\right)_{\square} = \frac{v_{\max} \cdot x_{50\%} \cdot y_{50\%}}{v_0 \cdot a \cdot b} \int \frac{v}{v_{\max}} \cdot d\left(\frac{x}{x_{50\%}}\right) \cdot d\left(\frac{y}{y_{50\%}}\right) = \frac{v_{\max} \cdot x_{50\%} \cdot y_{50\%}}{v_0 \cdot a \cdot b} T_{\square} \quad (7)$$

The volume flow rates in the different cross-sections of the jet are calculated using numerical integration, using the measured velocity values, as we can see later on in the guidelines.

3. Detailed description of the measurement task:

After the desired orifice is selected, ten velocity profiles have to be measured in various heights (z), which are provided below.

3.1 Installing the selected orifice in the test rig.

3.2 Selecting the proper measuring heights. Considering the highest point of the measuring stand and the level of the orifice (z=0), the following heights are recommended for measuring the velocity profiles along the diameters of the cross-sections:

z=0, 1d, 2.5d, 5d, 5.5d, 6d, 7d, 8d, 9d, 10d, where d corresponds to the diameter of the orifice. The minimum requirement is to measure the velocity profiles at 7 levels.

3.3 The pressure transducer has to be connected to the Prandtl probe using thin rubber tubes. In the case when the Betz manometer is used, we should perform the connection by carefully approaching the ends of the rubber tubes towards the taps of the manometer, whilst constantly checking the scale for changes. If a negative deviation is detected in the scale, the tubes are being attached to the improper taps, and this should be corrected.

3.4 The velocity profiles in the cross-sections $v(r)$ have to be determined as a function of the radius (r). For this, the Prandtl probe mounted to the measurement stand has to be moved to the desired positions, starting from one edge of the free jet and moving through the axis to the other edge with equidistant spacing being used between the measurement points. Near the orifice, in the initial section, the spacing between the measurement points should be 5 mm, and can be altered to 10 mm, 15 mm, and even 20 mm as the free jet becomes wider. Take care in setting the measurement stand, making sure to keep its zero point always in the geometrical axis of the free jet. At all the measurement heights it is important to know the position of the axis of the free jet compared to the axis of the outlet cross-section, since it may deviate by height.

After positioning the Prandtl probe to a certain measurement point, some 'settling time' is required before reading the values displayed by the pressure indicator, to give it time to settle to a relatively constant value. The measured values correspond to the local dynamic pressure in accordance with the operational principle of the Prandtl probe, by which the local velocity value can be calculated.

In every cross-section (different heights), the first step is to set the measuring stand by moving it to the axial position (r = 0), then measuring the velocity profile of the free jet from edge to edge, or in other words, as long as positive values are detected. Attention! It is not sufficient to measure one half of the free jet, as it may deviate at a greater distance from the orifice and thus its structure may become asymmetric.



4. Post-processing and Laboratory Report Guide

4.1 The following results should be included in the report:

The obtained pressure values in a tabular form. It is advisable to make separate tables for each height, which contain the registered pressure values as a function of the radius (r), the calculated velocity values and the total volume flow rate along with the parameters used for its calculation.

4.2 Diagrams have to be included:

The diagrams should be prepared by a suitable program, such as MS Excel, in which the required diagram type to select is the “xychart”. Take care in properly labeling and scaling the axis and providing the dimensions of all the plotted quantities.

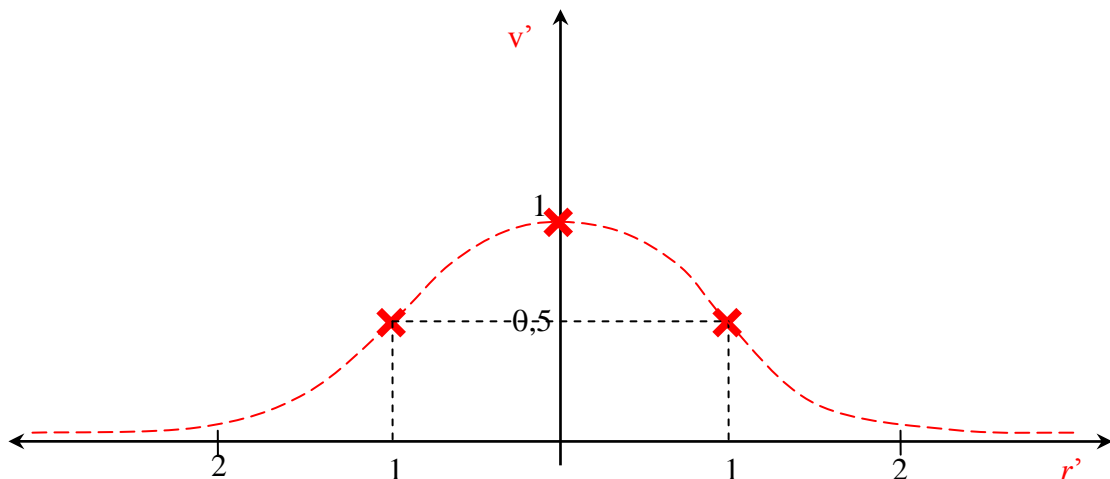
a) Plot all the velocity profiles (i.e.: $v(z,r) = f(z,r)$) in a **common** diagram

b) Plot the velocity profiles measured in the deceleration section (outside of the core distance) in another diagram (i.e.: $v(z,r) = f(z,r)$) if $z > z_m$.

c) To determine the core distance, plot the local maximum velocity values as a function of the height (z). (i.e. $v_{\max}(z) = f(z)$)

Hints: After plotting the maximum velocity values, fit a curve to them. Then intersect the curve at the value for 95 % of the outlet velocity with a horizontal line. Finally project this intersection to the x axis in order to obtain the z_{core} .

d) Plot the $z > z_{\text{core}}$ normalized velocity profiles (i.e. $v'(z,r') = f(z,r')$, where $v' = v_{\max}/v_0$ and $r' = r/r_{50\%}$). The normalized velocity profiles measured outside of the core distance should correspond with each other in case of proper data registration. In addition the plots have to be equal in the following points: ($r'=0$; $v'=1$), and ($r'=1$; $v'=0.5$) (See the figure below)



e) Plot the volume flow rate for each cross-section ($q_v = f(z)$).

The calculation of the volume flow rate should be performed as follows:

Divide the cross-section of the free jet into subsections in accordance with the position of the measuring points. In case of a cylindrical free jet, the most convenient division is to define semi-rings around each measuring point. (Except in the middle of the cross-



section, where a circle will be placed).. The thickness of the semi-rings is equal to the actual spacing. (5-10-15-20 mm) and the measurement points should be situated on the middle line of the corresponding ones. (*see the appendix*)

To determine the total volume flow rate simply add up the products of the semi-ring areas and the velocity values.

The data points of the volume flow rate plotted against the height (z) should not be linked. The transported volume flow rate should exhibit a linear connection with the height (z), and therefore a linear regression line starting from the value of the initial volume flow rate can be fit to the data points. It is practical to make a diagram, which includes the volume flow rates normalized by the initial volume flow rate (q_0), since it can be noticed at once how much the increment is as compared to the initial value.

4.3 Measurement error calculation

The absolute and relative errors of the velocity values measured for the point “k”:

$$v_k = \sqrt{\frac{2p_{din,k}}{\rho}} \quad \delta v_k = \sqrt{\sum_{i=1}^n \left(\delta X_i \cdot \frac{\partial v_k}{\partial X_i} \right)^2} \quad \frac{\delta v_k}{v_k} = ?$$

Where the Xi quantities and their errors are:

$$\begin{aligned} X_1 &= p_0, & \delta p_0 &= 100 \text{ Pa} \\ X_2 &= T_0, & \delta T_0 &= 1 \text{ K} \\ X_3 &= \Delta h, & \delta \Delta h &= 0.001 \text{ m} \end{aligned}$$

Or, for the EMB-001 digital pressure transducer:

$$X_4 = p_{din}, \quad \delta p_{din} = 2 \text{ Pa.}$$

Task:

Calculate the relative errors for one cross-section, and then plot them in a diagram, as a function of the radius (r) along with the velocity profile.

Remember that during the labs:

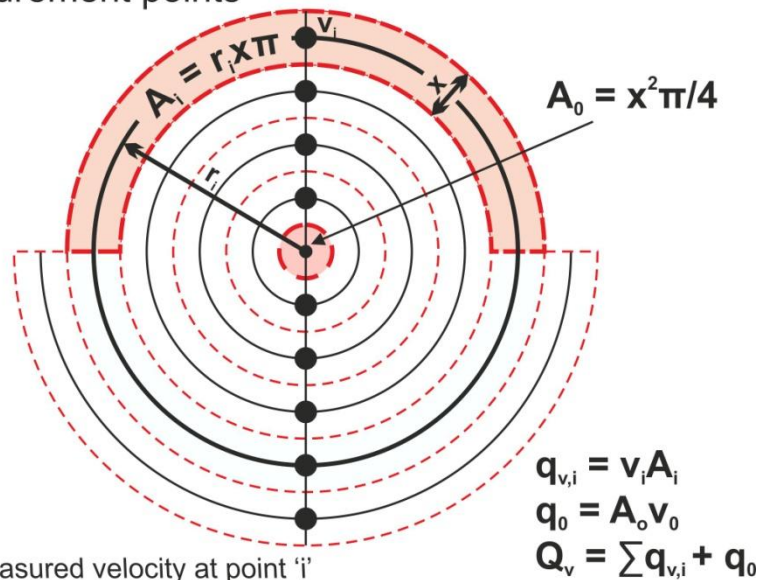
- ❖ Before turning any measurement device on, or in general during the lab, make sure that safe working conditions are ensured. The other participants have to be warned of the starting of the machines and of any changes that could endanger the members of the lab.
- ❖ The atmospheric pressure and room temperature should be recorded before and after every measurement. - The measurement units and other important factors (e.g. data sampling frequency, data of calibration) of every recorded value of the applied measurement devices should be recorded.
- ❖ Type and construction number of the applied measuring instrument should be included in the final report.
- ❖ Checking and harmonizing of the units of the recorded values with those used in further calculations.
- ❖ Manometers should be calibrated if necessary.
- ❖ The measurement ports of the pressure meter should be carefully connected to the correct pressure ports of the instrument.
- ❖ If inlet or outlet tubes are to be assembled with fans, connections should be airtight as escaping/entering air can significantly modify the measurement results



Appendix

M2. Numerical integration (midpoint method) of the volume flow rate at a circular cross-section

- measurement points



v_i [m/s] - measured velocity at point 'i'
 r_i [mm] - distance of point 'i' from the centre
 x [mm] - spacing between the measurement points