Technical Acoustics and Noise Control *(lecture notes)*

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Content:

7.1. Important composition of harmonic waves, (lecture notes)

The standing wave and the beat are important composition of harmonic waves with theoretic and practical importance. The goal is to build up a mathematic model to deepen our knowledge in wave acoustics, and to have the possibility making engineering calculation.

Standing wave:

The superposition of two harmonic wave of identical angular frequency (ω) and sound pressure amplitude $(\hat{p})$ traveling in opposite direction, creates standing wave.

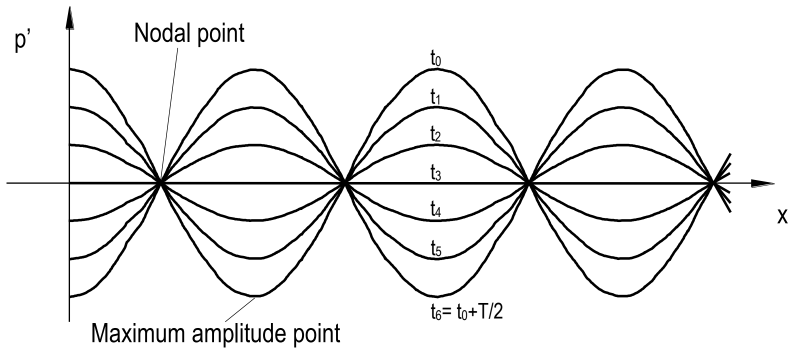
$$\hat{p}_1 = \hat{p}_2 \ , \quad \omega_1 = \omega_2 \ , \quad |a_1| = |a_2| \ , \quad a_1 \longrightarrow_{+x} \text{ és } a_{2\,-x}\longleftarrow$$

To derive the wave function we have to apply the principle of linear superposition, so the sound pressure of the resultant sound field, will be the simple algebraic sum of the sound pressures of the component sound fields.

$$p'_{sw}(x,t) = p'_1(x,t) + p'_2(x,t) = \hat{p}_1 \cos(\omega_1 t - k_1 x) + \hat{p}_2 \cos(\omega_2 t + k_2 x) =$$

With the application some trigonometric identity,

$$= \hat{p}\left(\cos(\omega t - kx) + \cos(\omega t + kx)\right) = 2\hat{p}\cos(\omega t)\cos(kx)$$



The graph of the sound pressure in a standing wave function versus distance in different time

Comments:

- In the argument of the wave function the t and x variables are separated, so the new occurrence is not a propagating wave.

- The standing wave is a one dimensional interference. Because of the interaction of the components, there are periodic amplification (loud) and destroying interference (silent) places along the x direction, that are constant in time.
- Important difference between the standing wave and the solution of the wave equation in closed space, at standing waves the amplitude only doubled and there are no unlimited number of series components.
- A standing wave will form, at perfect, perpendicular reflection of a harmonic wave.
- In the vicinity of sound reflecting surfaces the sound field of a source can create an undesired modification of sound pressure distribution. Close to sound reflecting surfaces measurements should complete carefully.

Beat:

The superposition of two harmonic wave of identical sound pressure amplitude ($\hat{p}$) and slightly different angular frequency (ω) traveling in the same direction, creates a beat.

$$\hat{p}_1 = \hat{p}_2 \ , \quad a_1 = a_2 \ , \quad \omega_1 > \omega_2 \ , \text{but} \quad \omega_2 \gg \omega_1 - \omega_2$$

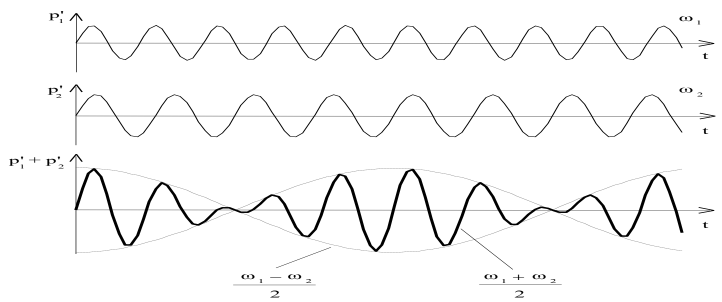
Let's apply the principle of linear superposition, and some trigonometric identity,

$$p'_l(x,t) = p'_1(x,t) + p'_2(x,t) = \hat{p}_1 \, cos(\omega_1 t - k_1 x) + \hat{p}_2 \, cos(\omega_2 t - k_2 x) =$$

$$= \hat{p} \left(cos(\omega_1 t - k_1 x) + cos(\omega_2 t - k_2 x) \right) =$$

$$= 2\hat{p} \, cos\left(\frac{\omega_1 + \omega_2}{2} t - \frac{k_1 + k_2}{2} x\right) cos\left(\frac{\omega_1 - \omega_2}{2} t - \frac{k_1 - k_2}{2} x\right)$$

The graphic sum of the component help us to understand the physical interpretation of the solution,



The sound pressure distribution of the components and resultant waves of a beat in time, at x= 0m position

Comments:

- The superposition heavily effects the wave function, it turns to a product of cosines, but both of cosine arguments contain the t and x variables together, so the beat left a propagating wave.
- The graphic sum shows the physical explanation how the beat is formed. At the beginning the components starting from the same phase, two positive maximums are added, but the small difference of angular frequency results after a long period of time an opposite phase state, and the components cancel each other, which periodically repeated in time.
- The base frequency of the beat is the simple mathematic average of the component frequencies, and that is modulated with the half of the bigger and smaller frequencies difference.
- In practise a standing wave could form, when for example two identical type of rotating machine working close to each other in a different operational point. The same type but different operational point slightly separates the rotational speed and frequency of the radiated noise.

- The subjective opinion of the periodically modulated pure tone is bad. So the beat can cause noise annoyance. On the other hand, the disturbing sound effect calls our attention, well separated from the usual background noise, so beat can use as a warning signal (fire service car, car anti-thief system, …).

7.2. Model testing of sound fields and similitude (lecture note)

Model test will be applied, when to investigate the problem in original size is impossible, dangerous or cost a lot of money. The model can be mathematic or physical. The mathematic model is a tool, to know occurrences without we do them in the real life. Selecting the variables and principles, creating equations and their solution, we can determine numerically the concrete measure what will happen. From an engineering approach the mathematics is a tool to design something. But never forget, analysing the result of the mathematic model, we can understand the described phenomena deeper, so the mathematic model serves the scientific understanding and research too. Physical modelling means, that the feature of the original occurrences will be investigated by experiments. The physical model will applied, when there is no mathematic model, or there is but the accuracy of the model is questioned. The physical models can be homologous or analogous. At homologous physical modelling the physical phenomena of the original and modelled cases are the same (for instance the flow around a car will be studied with a smaller size model in a wind tunnel). There are occurrences, which are difficult to measure experimentally, like to determine the water leakage in porous soil. In this case we should find a much more measurable, different phenomena occurrence, described by different variables, but the shape of the describing functions are the same in form, so they are analogous occurrences. In acoustics usually we do homologous models. It is very important that the original and the model occurrence have to be similar to each other. For example if in a certain place of an auditory hall the sound pressure distribution shows a non-desired elevation, it should turn out at the small size model too. The similarity can be determined by mathematic or physical condition. At the mathematic condition the original and the model occurrences are similar, when the dimensionless differential equations, boundary and initial conditions related to the occurrences are equal to each other. At homologous modelling the type of the equations are the same, so only the identity of the dimensionless constants is enough to the similarity. The condition based on physical approach, is the identity of the ratio at the original and the model case as well, calculated form the characteristic variables of the occurrences. For example the ratio of the characteristic variables for viscous fluid flow is ratio of the inertial force and frictional force (Reynold-number).

To determine the condition of the similarity in acoustics, based on mathematic approach, we should start from the homogeneous wave equation,

$$\frac{1}{a^2}\frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial x^2} = 0$$

To change the variables in the equation in a dimensionless form, both side of the equation let multiply with the characteristic length (L_0) square, and divide with the characteristic sound pressure (p'_0), and let expand the numerator and denominator in the first term with characteristic time (T_0) square. The dimensionless wave equation,

$$\frac{L_0^2}{a^2 T_0^2}\frac{\partial^2\left(\frac{p'}{p'_0}\right)}{\partial\left(\frac{t^2}{T_0^2}\right)} - \frac{\partial^2\left(\frac{p'}{p'_0}\right)}{\partial\left(\frac{x^2}{L_0^2}\right)} = 0$$

At homologous modelling the condition of the similarity is the identity of the coefficient in the first term left hand side. If this ration is constant, the square of it must constant too, and let focus on the harmonic waves, where the characteristic time is the time of period (T_0). The product of the speed of sound and the time of period is the wave length (λ), so the similarity conditions for sound fields are the identity of the ratio of the characteristic length and wave length, called Helmholtz-number (He),

$$He = \frac{L_0}{aT_0} = \frac{L_0}{\lambda_0}$$

Comments:

- In acoustics usually the models are smaller than the original sizes. Based on the derived similarity condition the relation between the characteristic length and wave length is proportional, and between the characteristic length and frequency is inverse proportional. In engineering noise control and room acoustics the upper limit of the frequency range of interest can reach even 10kHz. The real model scale is around 1:10, so the upper frequency of the microphone applied to the acoustic model measurement have to include the 100kHz, which leads lower sensitivity and higher price.

- Increasing the frequency another problem, the sound attenuation during the sound propagation, will appear. During sound propagation along pair meter distance till 1…2kHz frequency the energy loss can neglect. From 10kHz the attenuation because of dissipation turns to important and cannot neglect. So the dissipative losses can spoil the accuracy of the model test.

7.3. Test questions and solved problems

T.Q.1. Define the standing wave, derive its wave function, and explain its practical importance!

T.Q.2. What is the definition of the beat, derive its wave function, and explain its practical importance!

T.Q.3. What does mathematic and physical model mean, what are the condition of the similarity in homologous physical modelling?

S.P.1. A harmonic wave of 0.001 sec time of period and 0.5 Pa sound pressure amplitude perfectly reverberated from a perpendicular positioned, plane surface. Give the name of the acoustic occurrence, calculate the characteristic frequency, the distance between two nodal points and the minimum and maximum sound pressure amplitude of the resultant wave! The air temperature is 10°C.

Solution:

The name of this superposition is standing wave.

The frequency is f= 1/T= 1kHz,

The distance between two neighbouring nodal point is the half wave length,

The speed of sound: $a = \sqrt{\kappa R T_0} = \sqrt{1.4 \cdot 287 \cdot (273 + 10)} \approx 337.2\ m/s$

The wave length: $\lambda = aT \approx 337.2 \cdot 0.001 \approx 0.3372\ m$

The distance between two nodal points: $\lambda/2 \approx 0.3372/2 \approx 0.1686\ m$

The resultant sound pressure amplitude in the nodal (minimum) point is 0 Pa.

The resultant sound pressure amplitude in the nodal point is 0Pa, and in the anti-nodal (maximum) point is the simple sum of the component amplitudes, 1Pa.

S.P.1. The sound field of a theatre hall will be tested with homologous physical model. The geometric scale of the model is 1:10. Calculate the test sound frequencies, if the relevant frequencies of the sound in the original size are 250, 500 and 1000 Hz. The medium in the original and test case are air in normal state (p_0= 1bar, t_0= 20˚C) as well.

Solution:

$He_m = He_0$, $l_m/\lambda_m = l_0/\lambda_0$, $l_m f_m/a_m = l_0 f_0/a_0$, ($a_m = a_0$), $f_m = f_0\ l_0/l_m = 10 \cdot f_0$, the test frequencies: 2.5k, 5k and 10kHz

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