Budapesti University of Technology and Economics

Department of Fluid Mechanics

Pre-measurement class II.

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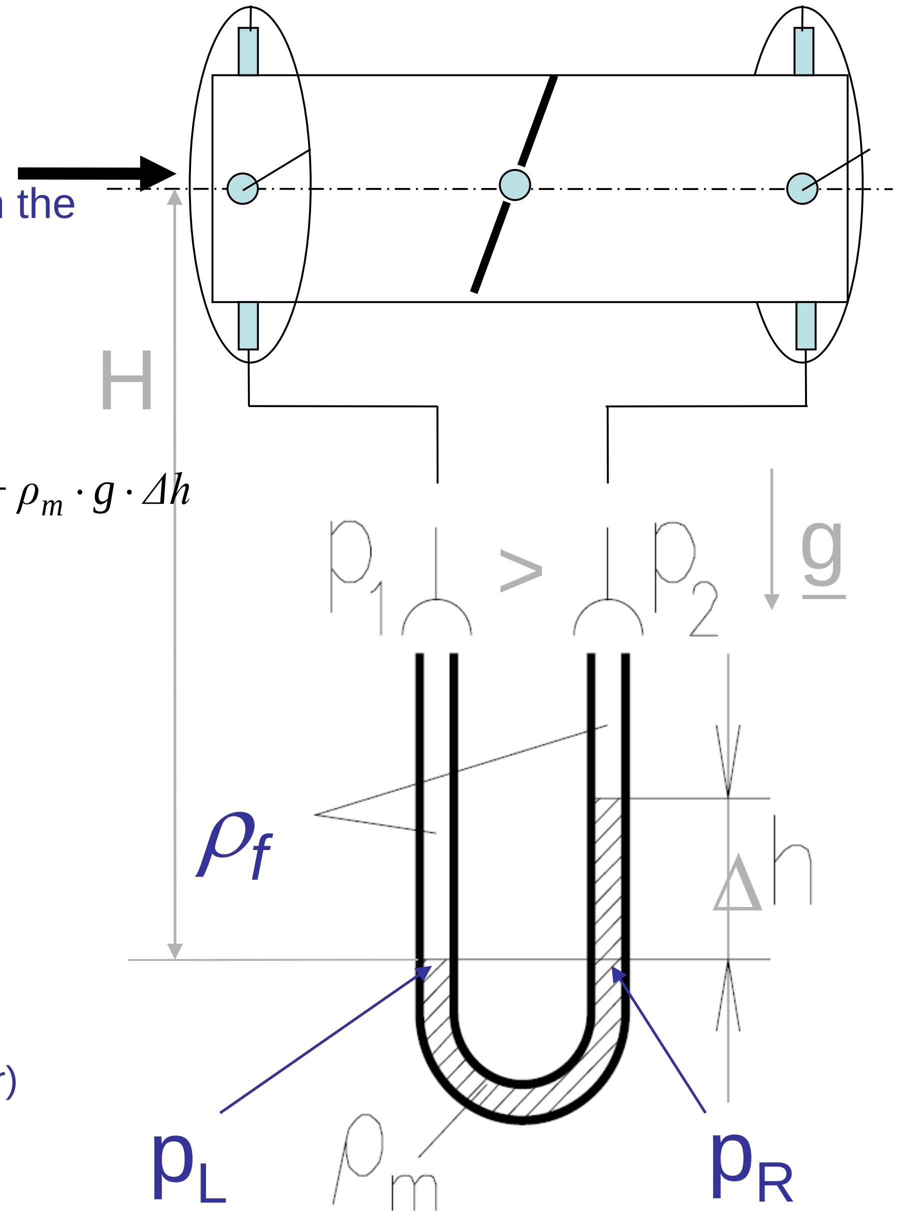
Measuring pressure differences (measuring Δp)

- Provides the basis of many measurements (e.g. velocity, volume flow rate)
- For a fluid medium, pressure differences can be measured between two points
- It is often measured with regard to a reference value (atmospheric pressure, static pressure in a duct)
- Measurement instruments
  - U tube manometer
  - Betz manometer
  - Inclined micro manometer
  - Bent tube micro manometer
  - EMB-001 digital handheld manometer

Measuring Δp / U tube manometer I.

- Pipe flow
- Butterfly valve
- Average of the pressure measured on the pressure taps around the perimeter



The manometers balance equation:

$$p_L = p_R$$

$$p_1 + \rho_f \cdot g \cdot H = p_2 + \rho_f \cdot g \cdot (H - \Delta h) + \rho_m \cdot g \cdot \Delta h$$

$$p_1 - p_2 = (\rho_m - \rho_f) \cdot g \cdot \Delta h$$

Can be simplified if

$$\rho_f << \rho_m$$

(e.g. if the measured fluid is air

and the measurement fluid is water)

$$p_1 - p_2 = \rho_m \cdot g \cdot \Delta h$$

Notice that $\Delta p \neq f(H)$

Measuring Δp / U tube manometer II.

The manometers balance equation:

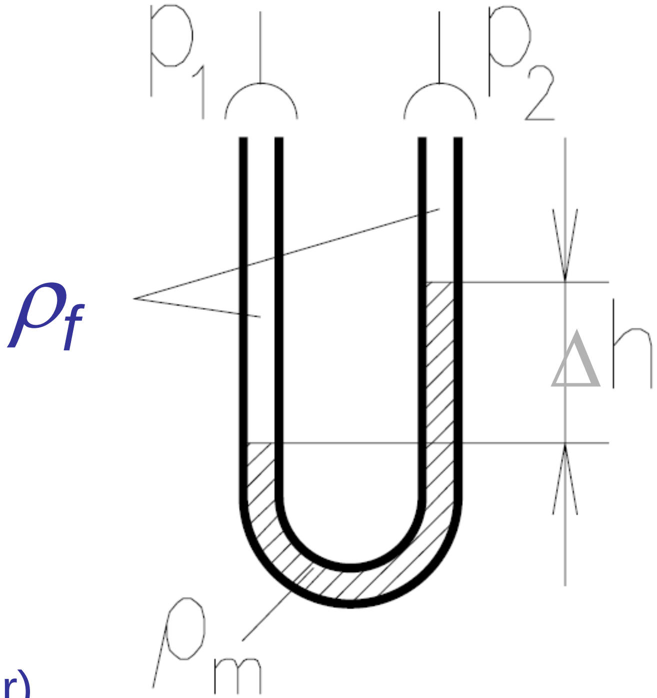
$$\Delta p = \left(\rho_m - \rho_f\right) g \Delta h$$

Density of the measuring fluid ρ_m (approximately)

$$\rho_{Hg} \approx 13600 \frac{kg}{m^3}$$ mercury

$$\rho_{water} \approx 1000 \frac{kg}{m^3}$$ water

$$\rho_{Alcohol} = 830 \frac{kg}{m^3}$$ alcohol



Density of the measured fluid: ρ_f (For example air)

$$\rho_{air} = \frac{p_0}{R \cdot T} = 1.19 \frac{kg}{m^3}$$

p_0 - atmospheric pressure [Pa] ~10^5Pa

R - specific gas constant for air 287[J/kg/K]

T - atmospheric temperature [K] ~293K=20°C

Measuring Δp / U tube manometer III.

Example: the reading: $$\Delta h = 10mm$$

The accuracy ~1mm: The absolute error:

$$\delta(\Delta h) = \pm 1\,mm$$

How to write the correct value with the absolute error(!)

$$\Delta h = 10mm \pm 1mm$$

The relative error:

$$\frac{\delta(\Delta h)}{\Delta h} = \frac{1\mathrm{mm}}{10mm} = 0.1 = 10\%$$

Disadvantages:

- Reading error (take every measurement twice)
- Accuracy~1mm
- For a small pressure difference, the relative error is large

Advantages:

- Reliable
- Does not require servicing

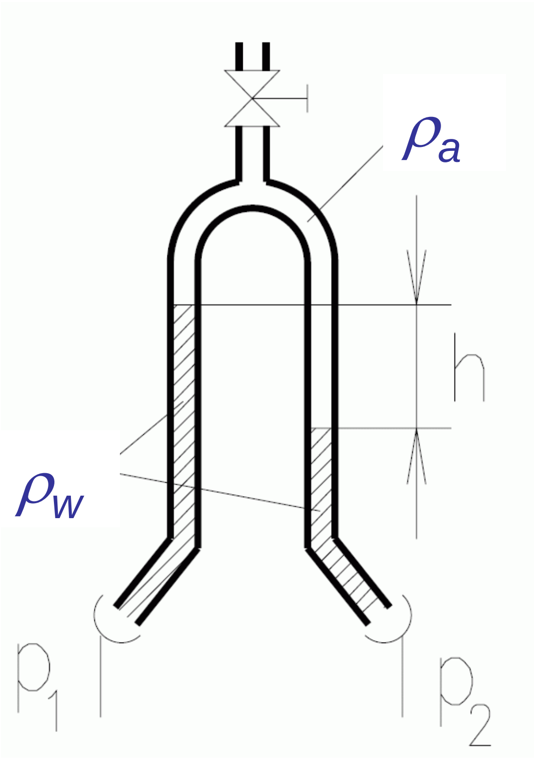
Measuring Δp / upside down U tube micro manometer

The manometer's balance equation

$$p_1 - p_2 = (\rho_{water} - \rho_{air}) \cdot g \cdot h$$

Since in most cases upside down U tube manometers are used to measure liquid (e.g. water) filled lines, the measurement fluid is usually air, and the density ratio is therefore (1.2/1000). -ρ_a (density of air) can be neglected.

The advantage of this measuring device is that when it is used for liquid filled systems, air can be used instead of mercury in order to improve the accuracy of the relative error of the readings!



Measuring Δp / Betz micro manometer

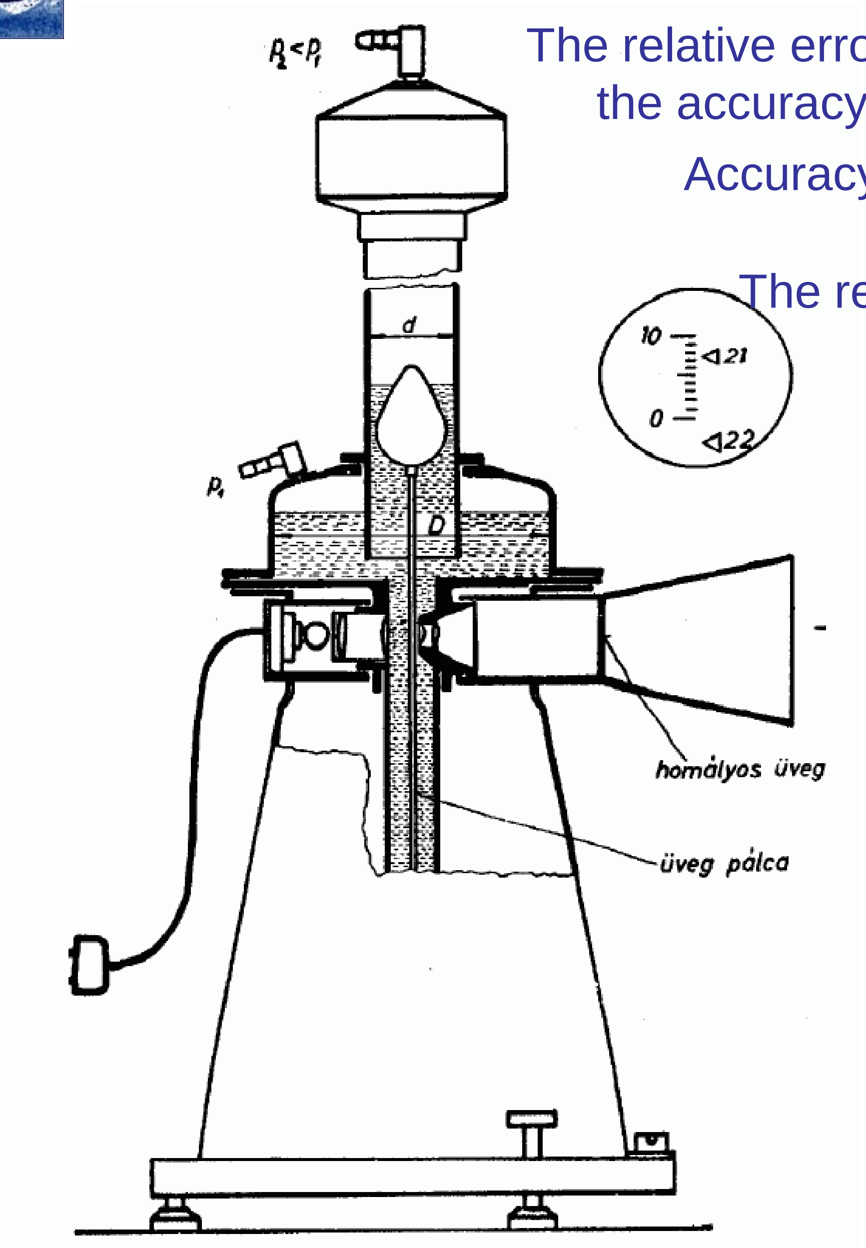
The relative error is reduced by optical means, improving the accuracy.

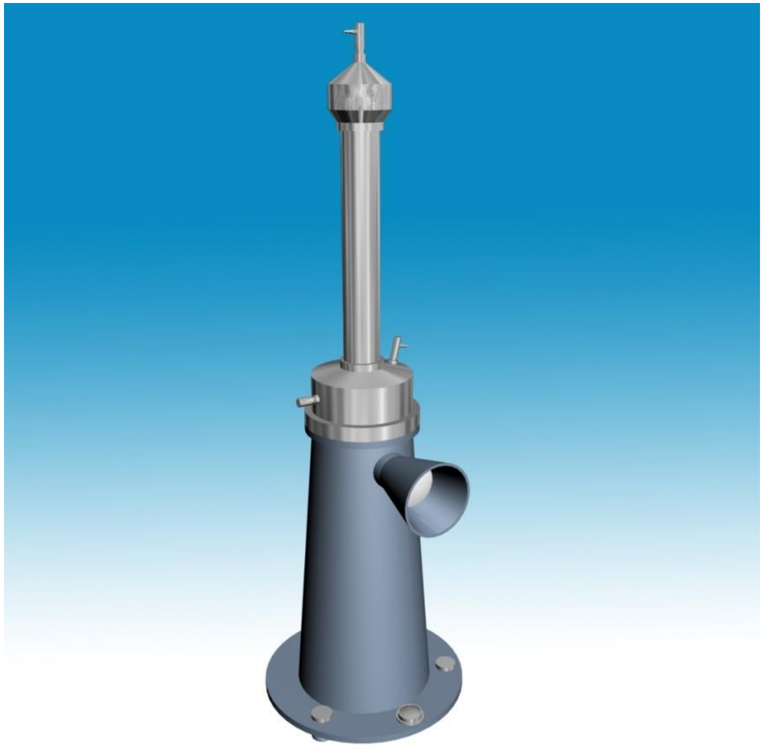
Accuracy ~0,1mm: The absolute error is:

$$\Delta h = 10mm \pm 0.1mm$$

The relative error:

$$\frac{\delta(\Delta h)}{\Delta h} = \frac{0.1mm}{10mm} = 0.01 = 1\%$$



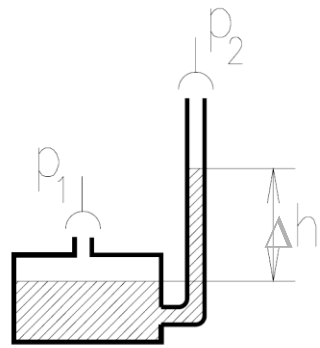


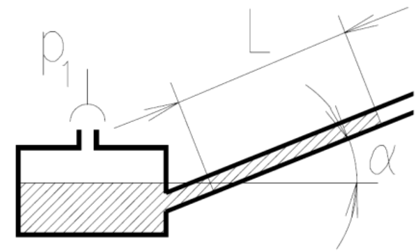
Measuring Δp / inclined micro manometer

The manometers balance equation

$$p_1 - p_2 = \rho_m \cdot g \cdot \Delta h$$

$$\Delta h = L \cdot \sin\alpha$$

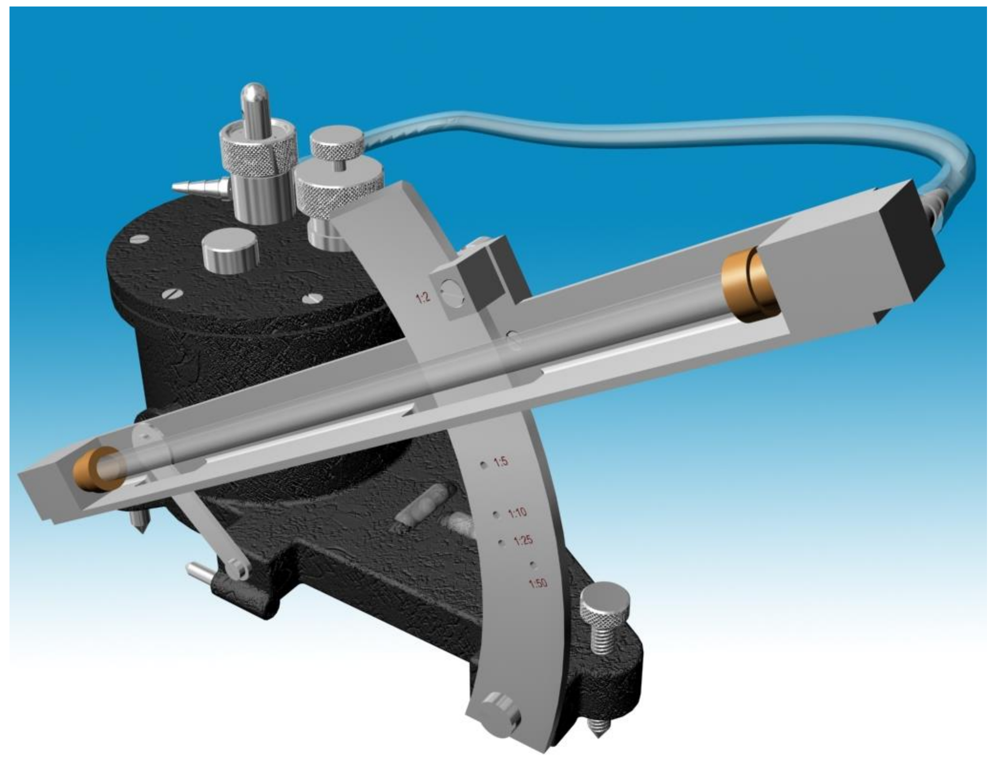




Accuracy: δL~±1mm,

Relatív error in the case of α=30°

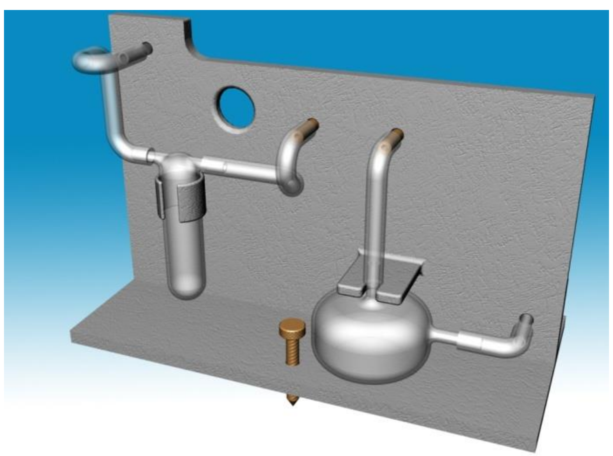
$$\frac{\delta L}{L} = \frac{\delta L}{\frac{\Delta h}{\sin\alpha}} = \frac{1mm}{\frac{10mm}{\sin 30°}} = 0.05 = 5\%$$

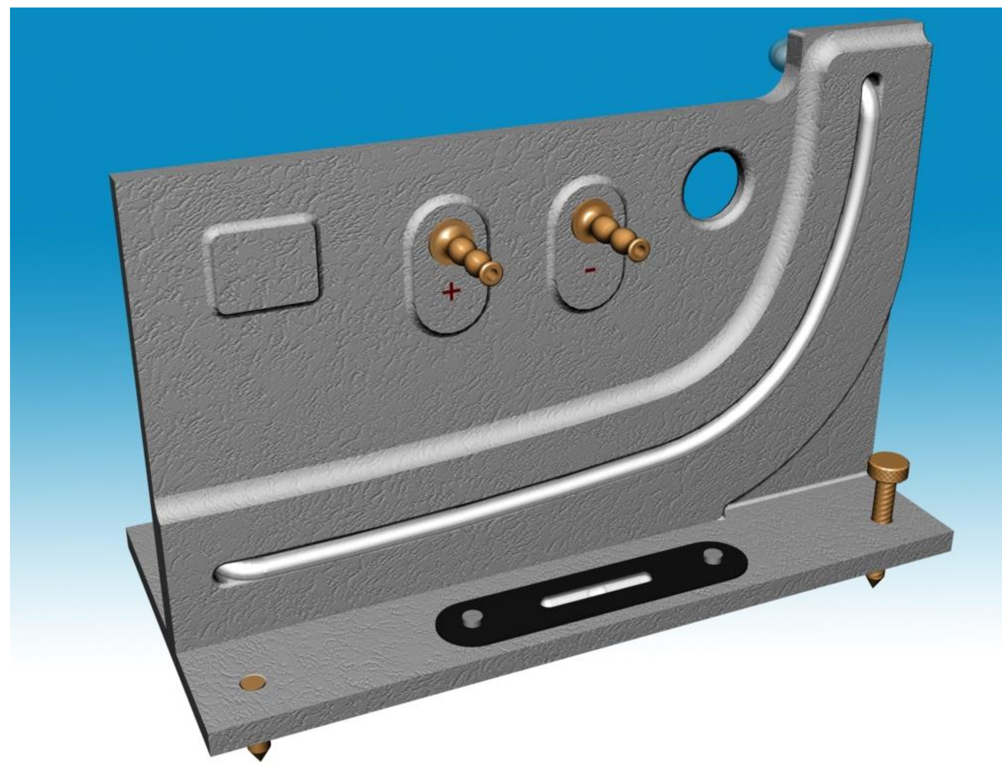


The relative error is a function of the inclination angle - $f(\alpha)$ - It is characterized by a changing relative error.

Measuring Δp / bent tube micro manometer

Is characterized by a
constant relative error
and a nonlinear scale





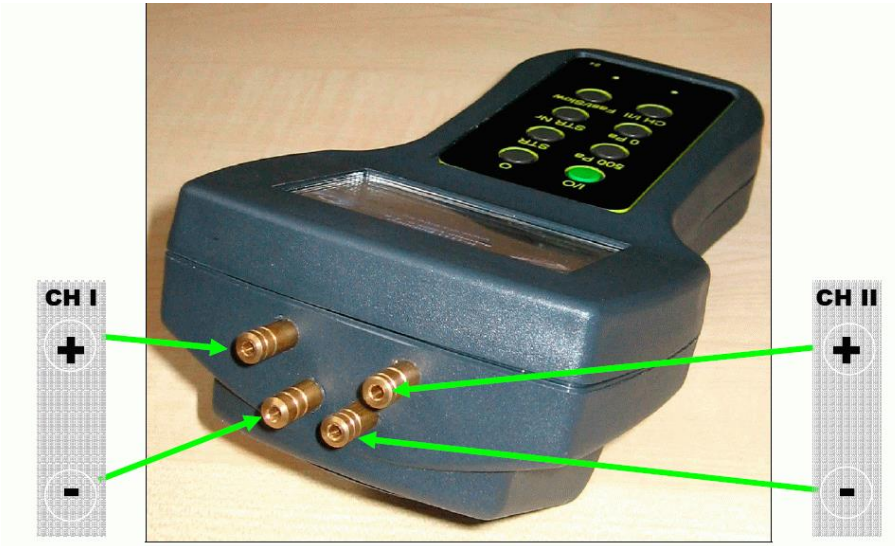
Measuring Δp / EMB-001 digital manometer

List of buttons to be used during the measurements

On/Off	Green button
Factory reset	„0” followed by the „STR Nr
Changing the channel	„CH I/II”
Setting 0 Pa	„0 Pa”
Averaging time(1/3/15s)	„Fast/Slow” (F/M/S)

Measurement range: $\Delta p = \pm 1250 Pa$

Measurement error: $\delta \Delta p = 2 Pa$



Measuring Δp / EMB-001 digital manometer

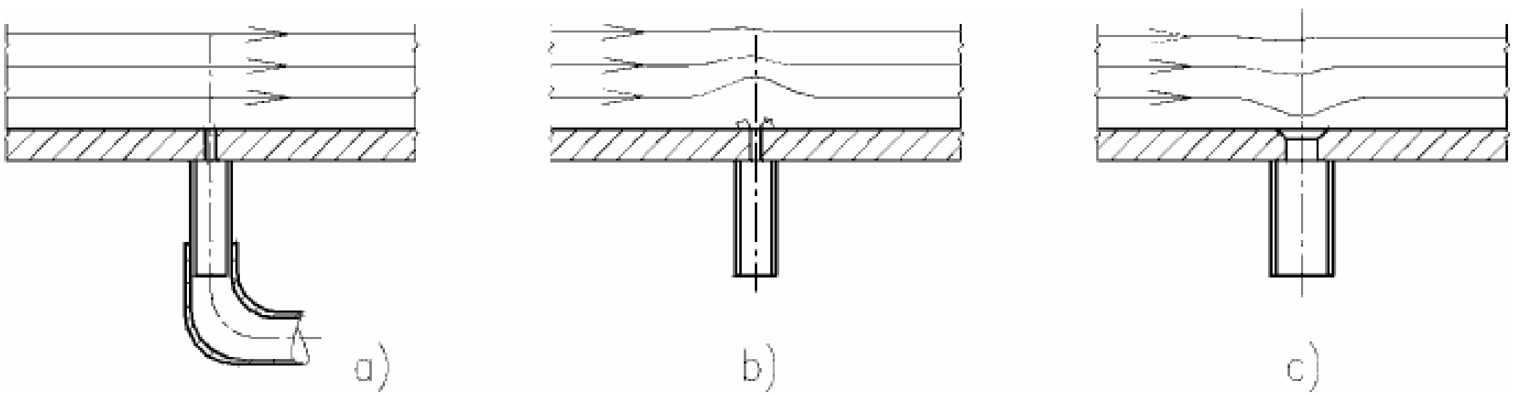
During the course of the laboratory sessions, the digital manometers need to be calibrated to the Betz manometers.

Measuring Δp / Pressure tap

When measuring pressures we need the streamlines to be parallel and straight

In this case the pressure is not changing perpendicularly to the streamlines

(The normal component of the Euler equation)



a) Correct b) c) Incorrect

Velocity measurement devices

- Pitot tube/probe
- Pitot-static (Prandtl) tube/ probe

Velocity measurement / Pitot tube/probe

Pitot, Henri (1695-1771), French engineer.

Determining the dynamic pressure:

$$p_d = p_t - p_{st}$$

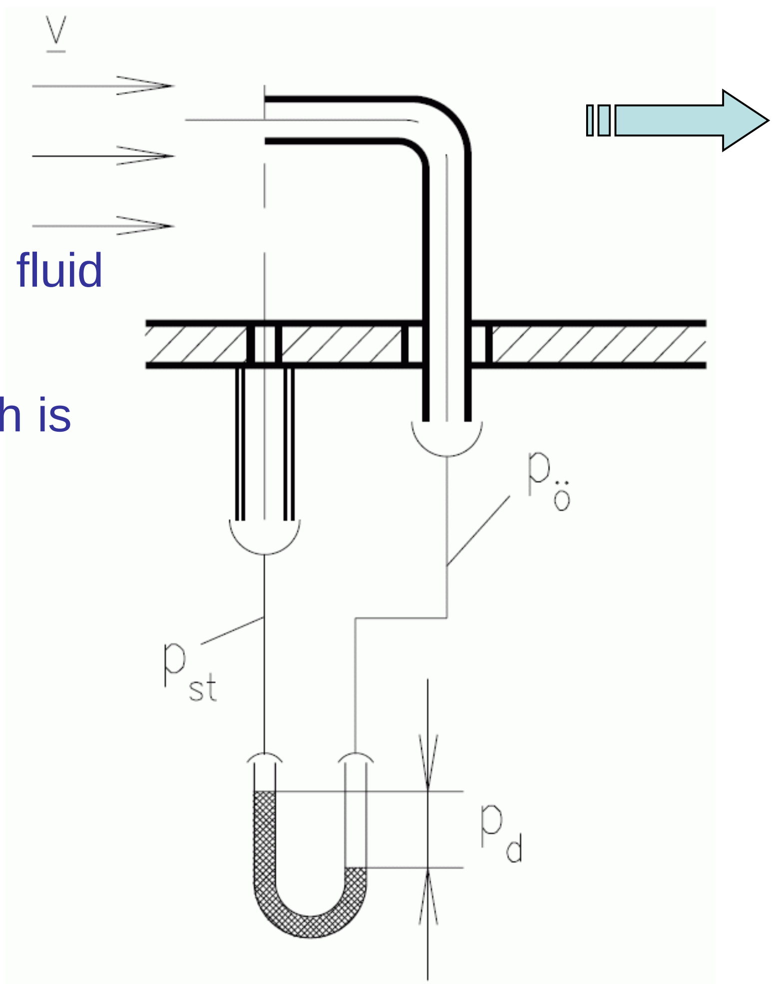
p_t the pressure measured in the stopped fluid (total pressure)

p_{st} the pressure acting on a surface which is parallel to the flow (static pressure)

$$p_d = \frac{\rho_f}{2} \cdot v^2$$

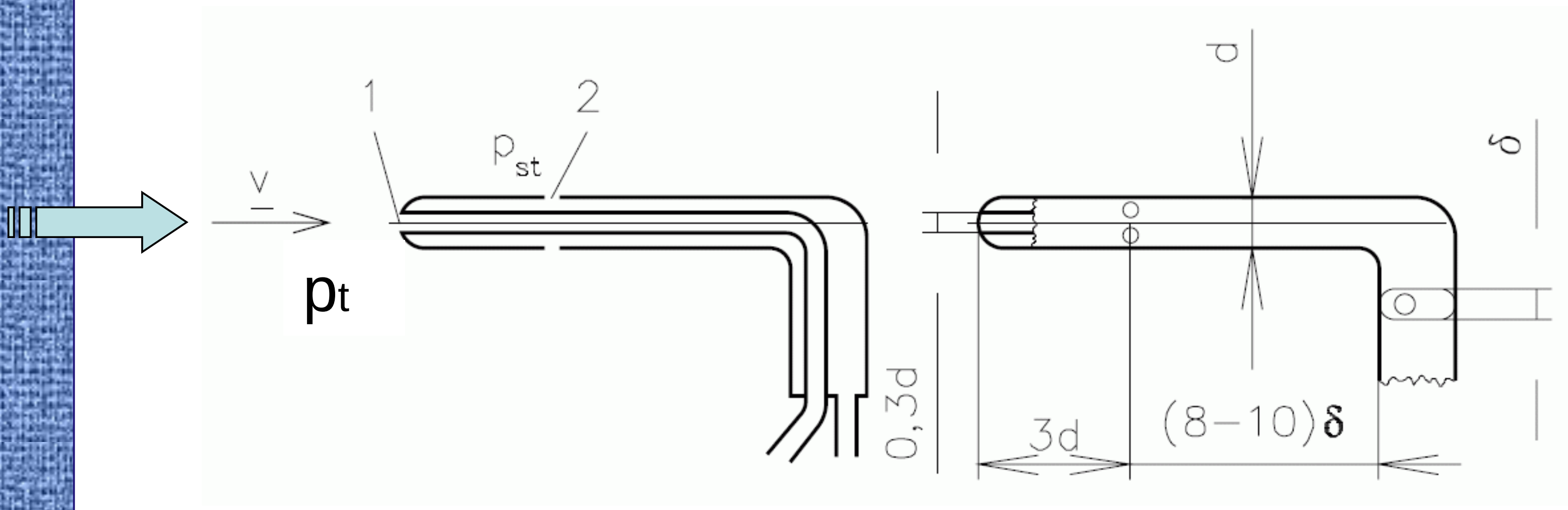
Determining the velocity:

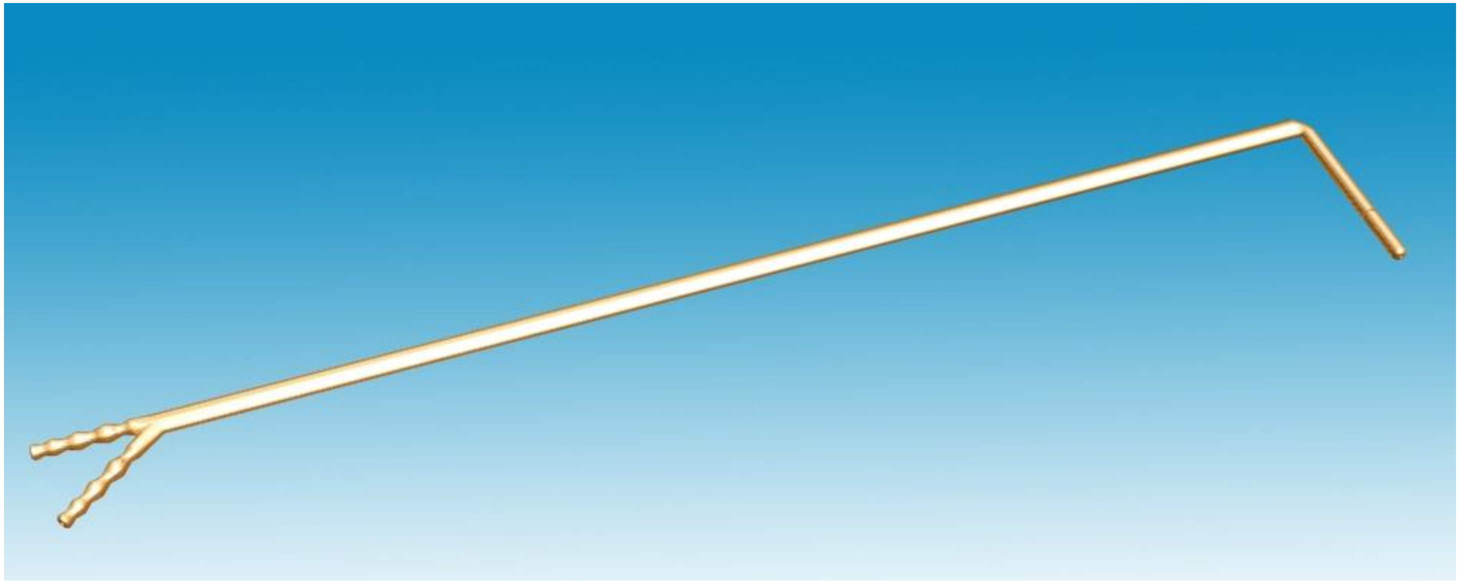
$$v = \sqrt{\frac{2}{\rho_f} \cdot p_d}$$



Velocity measurement / Pitot-static (Prandtl) tube/probe

Prandtl, Ludwig von (1875-1953), German fluid mechanics researcher





Measuring volume flow rate

- Definition of volume flow rate
- Measurement method based on velocity measurements in multiple points
  - Non-circular cross-sections
  - Circular cross-sections
    - 10 point method
    - 6 point method
- Pipe flow meters based on flow contraction
  - Venturi flow meter (horizontal/inclined axis)
  - Through flow orifice (contraction ratio, iteration)
  - Inlet orifice
  - Inlet bell mouth

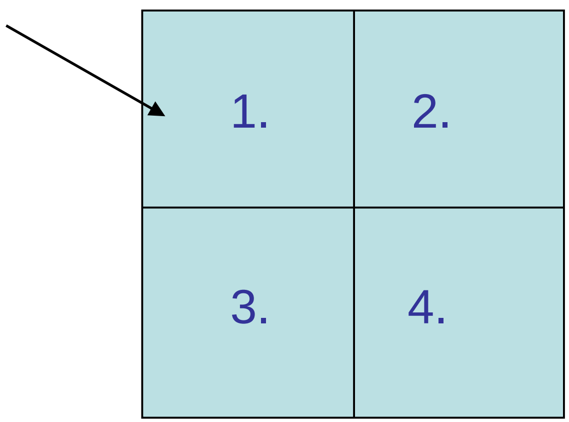
Calculating an average velocity from multiple velocity measurements

Very important: the square root of the averages ≠ the average of the square roots(!)

Example: Measuring the dynamic pressure in multiple points and calculating the velocity from it

$$v_i = \sqrt{\frac{2}{\rho_f} \cdot \Delta p_i}$$

$$v_1 = \sqrt{\frac{2}{\rho_f} \cdot \Delta p_1}$$



$$\bar{v} = \frac{\sqrt{\frac{2}{\rho_f} \cdot \Delta p_1} + \sqrt{\frac{2}{\rho_f} \cdot \Delta p_2} + \sqrt{\frac{2}{\rho_f} \cdot \Delta p_3} + \sqrt{\frac{2}{\rho_f} \cdot \Delta p_4}}{4}$$

Correct

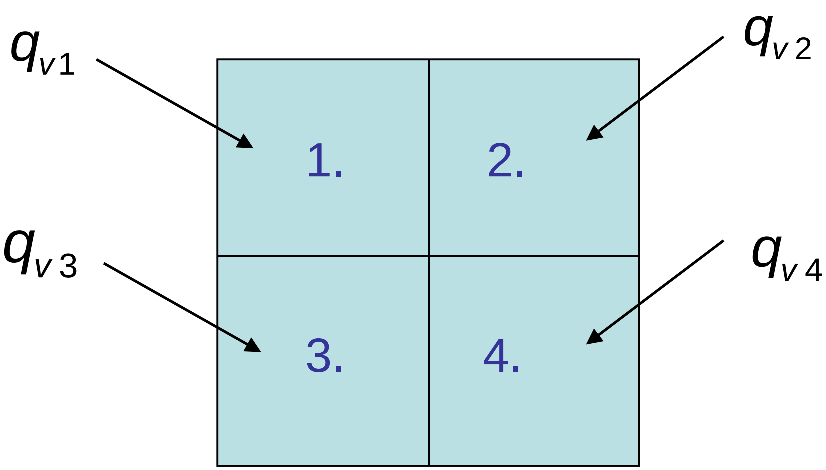
Volume flow rate / based on velocity measurements I.
Non-circular cross-sections

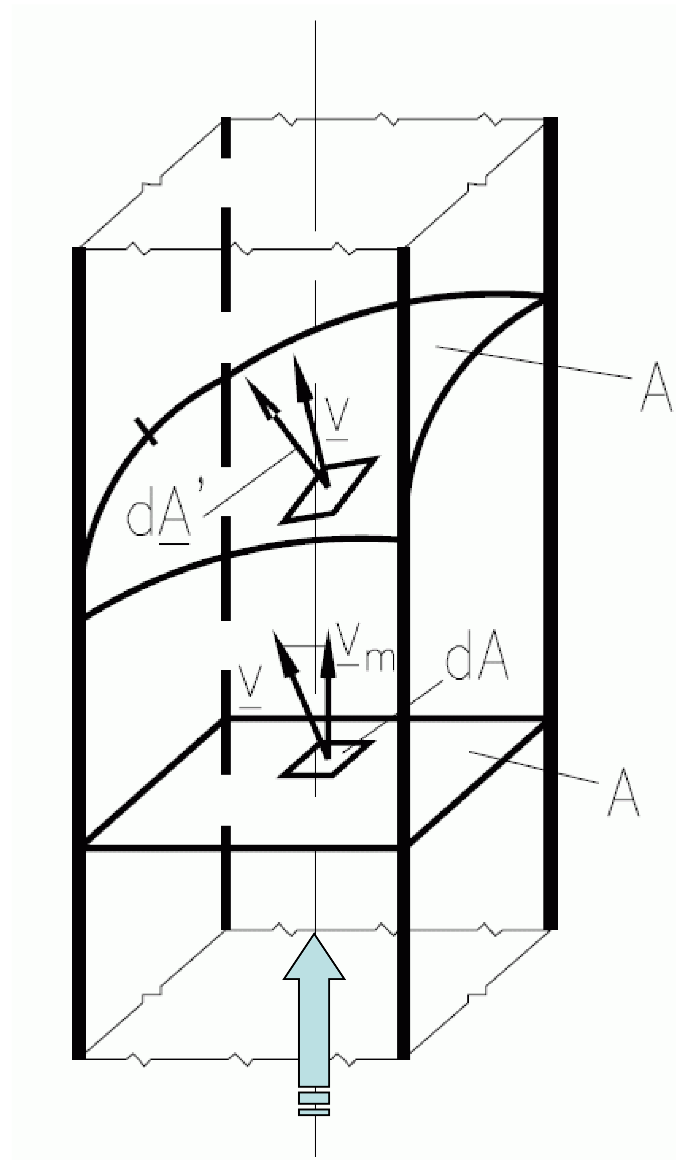
$$q_v = \int_A \underline{v} \cdot \underline{dA} \approx \sum_{i=1}^{n} v_{m,i} \cdot \Delta A_i$$

Assumptions:

$$\Delta A_1 = \Delta A_2 = \Delta A_i = \frac{A}{n}$$

$$q_v = \Delta A_i \cdot \sum_{i=1}^{n} v_{m,i} = \frac{A}{n} \cdot \sum_{i=1}^{n} v_{m,i} = A \cdot \bar{v}$$

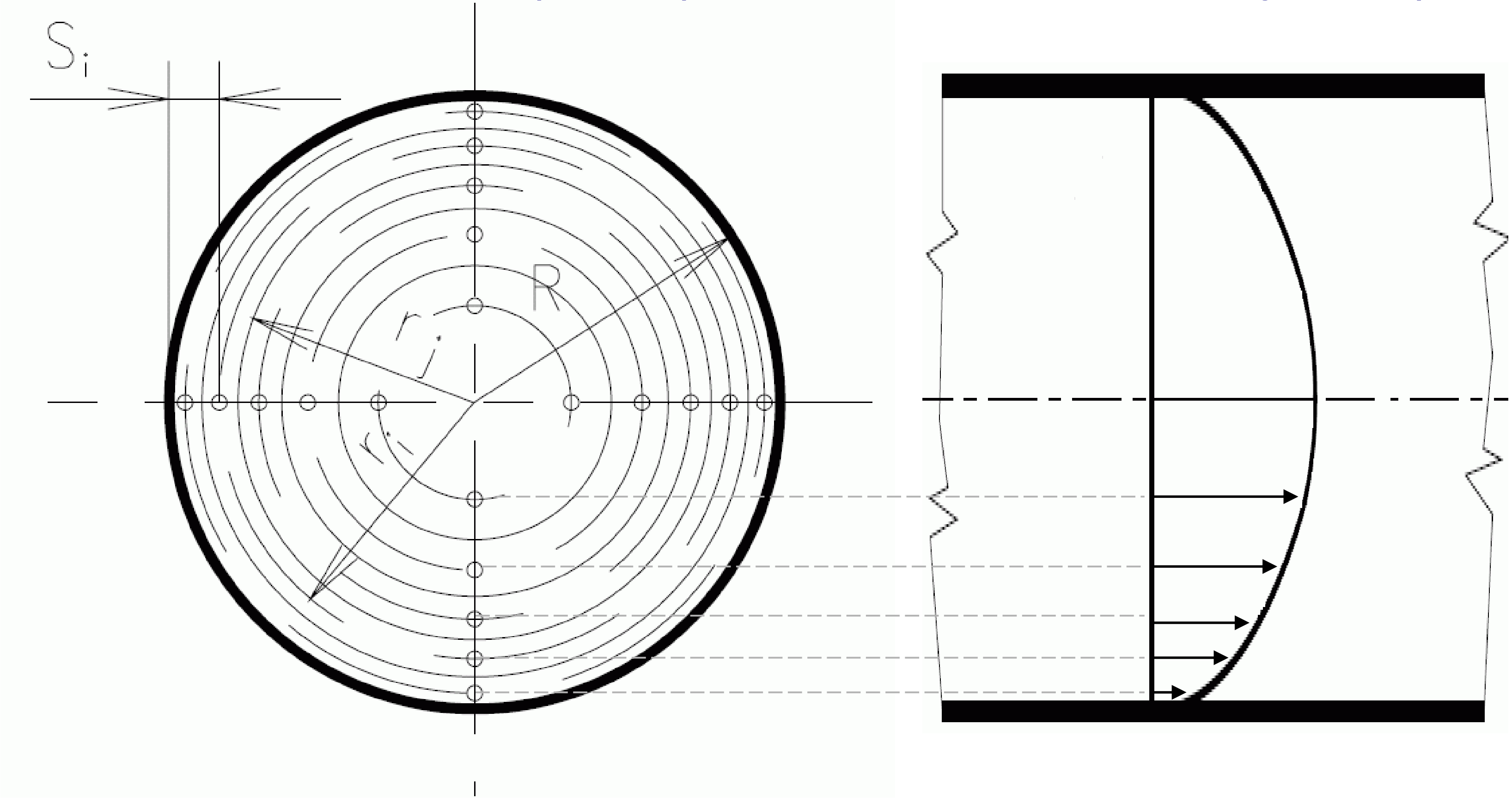




Volume flow rate / based on velocity measurements II.

Circular cross-sections, 10 point (6 point) method

- The velocity profile is assumed to be a 2nd order parabola
- Steady flow conditions
- Based on Pitot-static (Prandtl) tube measurements of the dynamic pressure



This is a standardized procedure, and the measurement point are given in the standard (MSZ 21853/2):

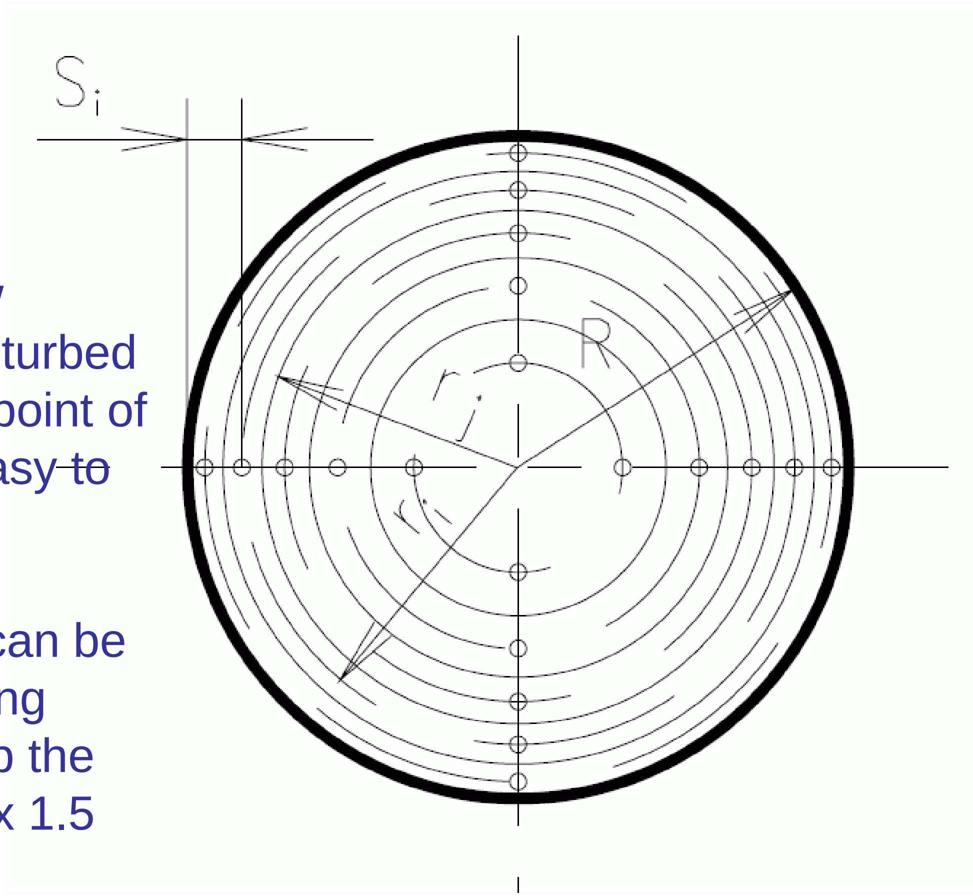
S_i/D= 0.026, 0.082, 0.146, 0.226, 0.342, 0.658, 0.774, 0.854, 0.918, 0.974

Volume flow rate / based on velocity measurements III.
Circular cross-sections, 10 point (6 point) method

$$q_v = A \cdot \frac{v_1 + v_2 + \ldots + v_{10}}{10}$$

Assumptions:

$$A_1 = A_2 = \ldots = A_{10}$$



The **advantage** of this method, as compared to methods based on flow contraction, is that the flow is not disturbed greatly, and therefore the operation point of the system is not altered, and it is easy to execute the measurements.

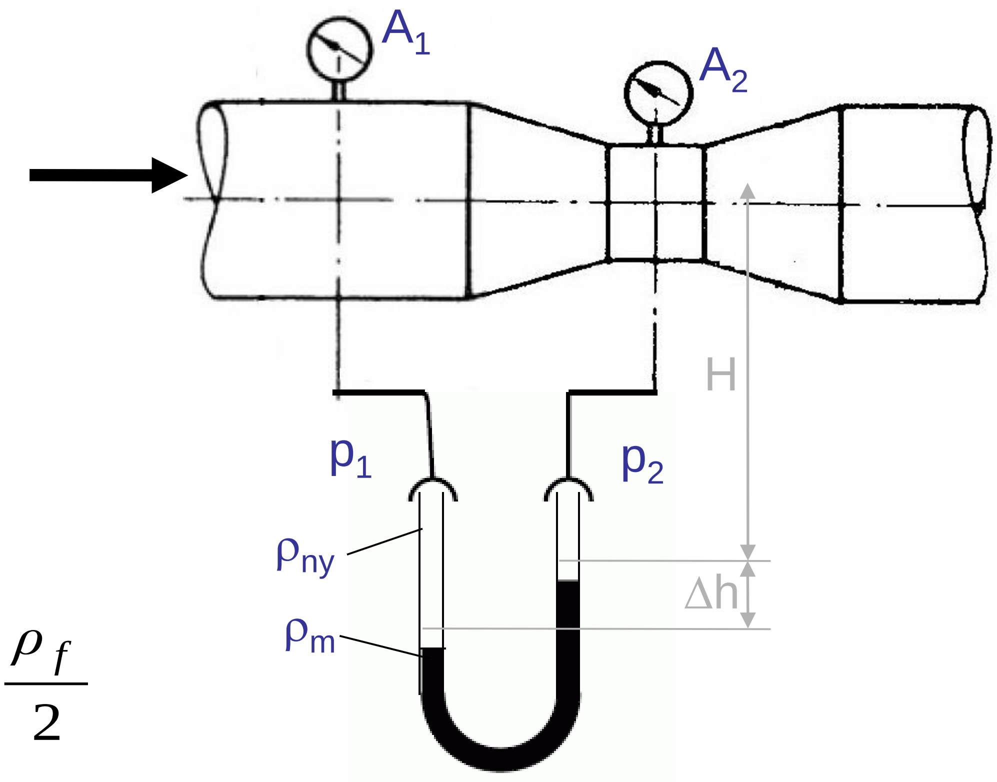
The **disadvantage** is that the error can be much larger with this method. For long measurements it is also hard to keep the flow conditions constant. (10 points x 1.5 minutes = 15 minutes)

Volume flow rate / flow contraction methods
Venturi pipe

If compressibility is negligible (ρ=constant):

$$q_v = v \cdot A = cont. \qquad [q_v] = \frac{m^3}{s}$$

$$q_v = v_1 \cdot A_1 = v_2 \cdot A_2$$



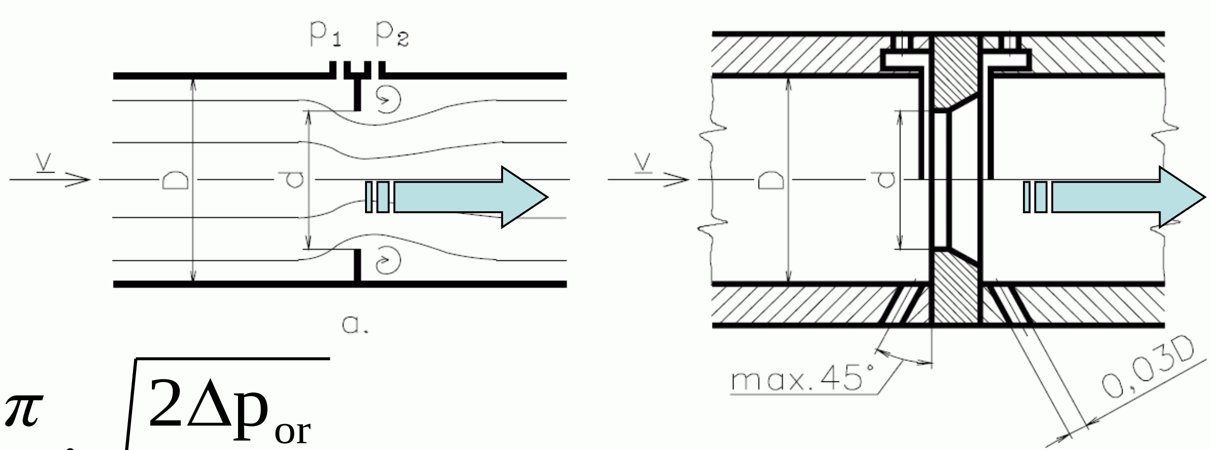
Bernoulli equation (ρ=const., U=const., no losses):

$$p_1 + v_1^2 \cdot \frac{\rho_f}{2} = p_2 + v_2^2 \cdot \frac{\rho_f}{2}$$

$$v_1 = \sqrt{\frac{(\rho_m - \rho_f) \cdot g \cdot \Delta h}{\frac{\rho_f}{2} \cdot \left[\left(\frac{d_1}{d_2}\right)^4 - 1\right]}} = \sqrt{\frac{\Delta p}{\frac{\rho_f}{2} \cdot \left[\left(\frac{d_1}{d_2}\right)^4 - 1\right]}}$$

Volume flow rate / flow contraction methods
Through flow orifice

Standard orifice - pressure difference



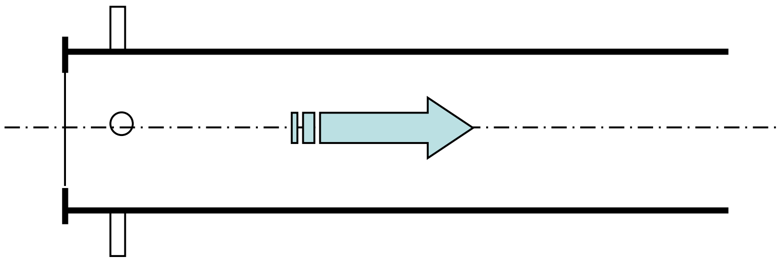
$$q_v = \alpha \cdot \varepsilon \cdot \frac{d_{or}^2 \cdot \pi}{4} \cdot \sqrt{\frac{2\Delta \mathrm{p_{or}}}{\rho}}$$

$\beta = d/D$	Cross-section ratio
d [m]	Diameter of the smallest cross-section
D [m]	Diameter of the pipe upstream of the orifice
$Re_D = Dv/\nu$	Reynolds number’s basic equation
v [m/s]	The average velocity in the pipe of diameter D
ν [m^2/s]	kinematic viscosity
p_1 [Pa]	The pressure measured upstream of the orifice
p_2 [Pa]	The pressure measured downstream of the orifice
ε	Expansion number ($\varepsilon=\varepsilon(\beta,\tau,\kappa)$~1 since for air, the change in pressure is small)
α	Contraction ratio, $\alpha=(\beta,Re)$ (When used according to the standard)
κ	Heat capacity ratio or Isentropic expansion factor
$\tau=p_2/p_1$	Pressure ratio

Volume flow rate / flow contraction methods
Inlet orifice (not standard)

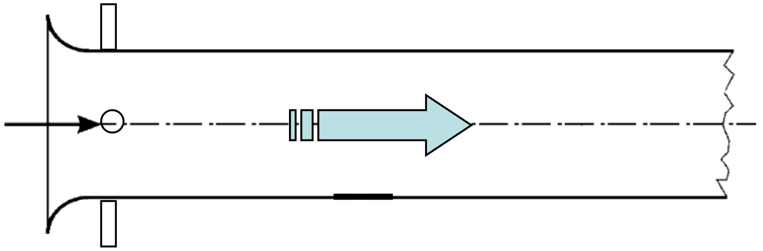
Not a standard contraction – pressure difference

$$q_v = \alpha \cdot \varepsilon \cdot \frac{d_{or}^2 \cdot \pi}{4} \cdot \sqrt{\frac{2\Delta \mathrm{p}_{\mathrm{or}}}{\rho}}$$



$$\alpha = 0.6$$

$$q_v = k \cdot \frac{d_{or}^2 \cdot \pi}{4} \cdot \sqrt{\frac{2\Delta \mathrm{p}_{\mathrm{or}}}{\rho}}$$



Downloadable material

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Researcher’s night

Measurement test: see materials online

Registration for measurements