Example problems for BSc laboratory measurements

1. Describe the calculation of air density based on atmospheric pressure and temperature! Give the names and units of the variables in the formula!

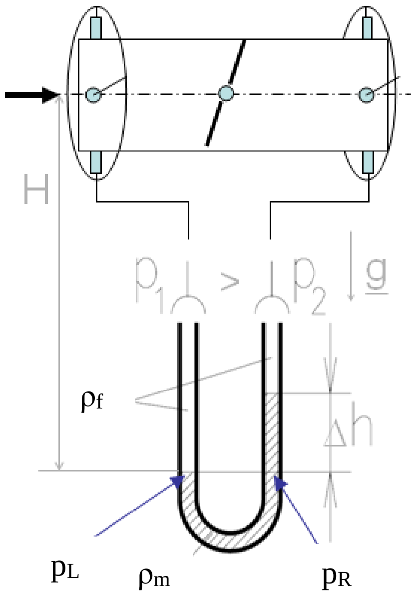
In order to determine air density, we use the ideal gas law. This is rearranged to get the expression for density:

$$\boxed{\rho = \frac{p_0}{RT}}, \text{ where:}$$

ρ: air density$[\rho] = kg / m^3$;
p_0: atmospheric pressure $[p_0]$ = Pa;
R: specific gas constant of air [R] = J / kg / K;
T: air temperature [T] = K.

2. Describe the U-tube manometer in a few sentences! How can you calculate the pressure from the level difference, if the U-tube is filled with a measuring fluid of density ρ_m, and it is connected to a pipe. There is water flowing in the pipe and a butterfly valve causes pressure loss between the two measurement points (hint: draw a figure). Give the names and units of all the variables in the equations!

The manometer has a U-shaped glass tube filled with a measuring fluid (ρ_m) that does not mix with the fluid of the flow (ρ_f), whose pressure is measured. The measuring liquid is usually water, mercury or alcohol.
The balance equation of the manometer is written for the lower interface between the fluids, where pressures in the two branches are equal:



$$p_L = p_R$$
$$p_1 + \rho_f \cdot g \cdot H = p_2 + \rho_f \cdot g \cdot (H - \Delta h) + \rho_m \cdot g \cdot \Delta h$$
$$p_1 - p_2 = (\rho_m - \rho_f) \cdot g \cdot \Delta h$$
$$\Delta p = (\rho_m - \rho_f) \cdot g \cdot \Delta h$$

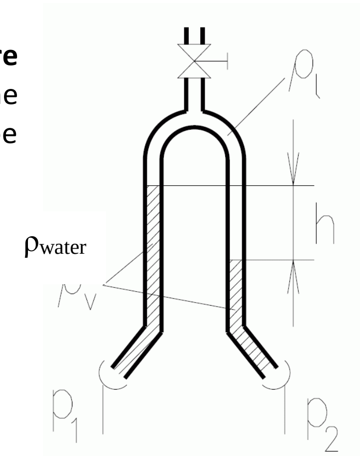
3. When do we use it and how does the upside-down U-tube manometer work?

Application:

It is usually applied in pipes filled with liquids, therefore the measuring fluid can be air, too. It's advantage is that it can measure **low pressure differences** much more precisely than a mercury-filled U-tube manometer, therefor the **relative error is reduced.**

How does it work?

It contains **two** (glass) **tubes** that are **connected to the pressure taps on the bottom** and are connected **together on top**. The amount of air on top (thus the height of the liquid level) can be set **using a tap**.



4. List and describe the methods that reduce the relative errors in case of manometers!

1.) One-time level reading

The U-tube manometer has the disadvantage that for every pressure reading, two heights have to be recorded. It can be avoided by replacing one of the branches with a tank of relatively large cross section, in which the fluid level change can be neglected.

2.) Using optical methods (Betz manometer)

Fluid level height can be determined more accurately using optical systems. In case of the Betz manometer, 0,1 mm of height difference can be read, that means 0,981 Pa uncertainty. It is 10% of the uncertainty of a U-tube manometer.

3.) Increasing fluid displacement for the same pressure

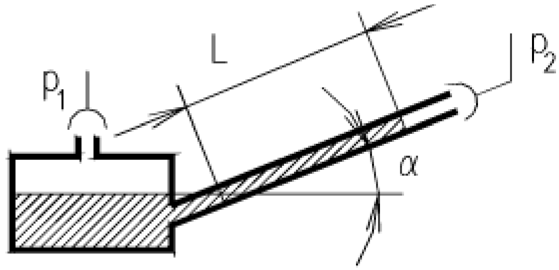
- **Reducing measuring fluid density**

By using a manometer filled with alcohol instead of water, the displacement increases by about 40% due to the lower density of alcohol. A drawback is that alcohol can absorb moisture from the air, therefore the density of the measuring fluid might change in time.

In case of **water flow and small pressure changes, an upside down U-tube manometer** can be applied instead of a conventional mercury-filled U-tube manometer. The error is reduced significantly.

- **Inclining a manometer branch**

In case of an inclined manometer tube the fluid displacement is $L = H / \sin(\alpha)$, therefore higher than in a vertical case.



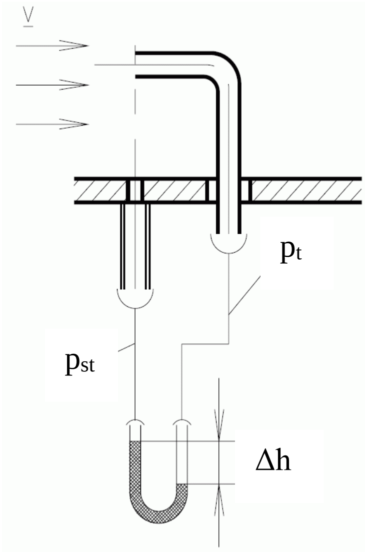
5. Define static, dynamic and total pressure! Give a formula, if relevant, together with names and units of the variables! Explain how these quantities can be measured!

Static pressure: pressure in the undisturbed flow. Symbol: p_∞, $[p_\infty]$ = Pa

Total pressure: a stagnation point pressure (pressure of the stopped fluid), symbol: p_t, $[p_t]$ = Pa

Dynamic pressure: the difference of the two above:

$p_d = \frac{\rho}{2} v_\infty{}^2$, where v_∞ is the undisturbed flow velocity, ρ is the density, $[p_d]$ = Pa



Static pressure can be measured using wall bores or pressure taps. Total pressure is measured using a Pitot tube, while dynamic pressure is the difference of these two. The streamlines must be parallel to each other and not curved. Otherwise a Pitot-static (Prandtl) probe is to be applied.

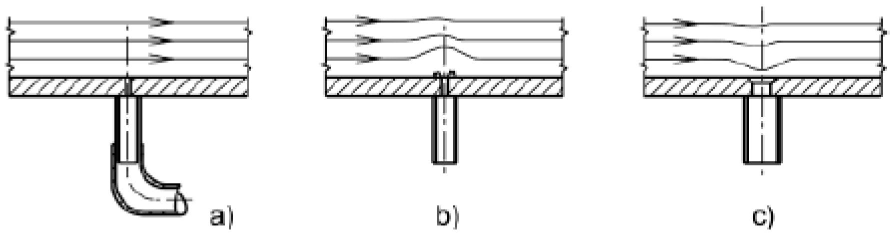
$p_d = p_t - p_{st}$

6. How can the static pressure be measured? How should one measure the pressure in case of flow in a pipe?

When measuring static pressure it is very important that the pressure taps do not influence the flow, e.g. do not bend the streamlines, so the taps cannot be rounded, chamfered and burr has to be removed, too.

Measurement: a wall bore of diameter 0.2-0.5 mm is to be made and a larger piece of tube has to be attached to it from the outer side. Plastic tubing can be attached onto this tube that leads the pressure to the manometer. There should not be any bubbles left in the tubing.

a) correct
b) incorrect
c) incorrect

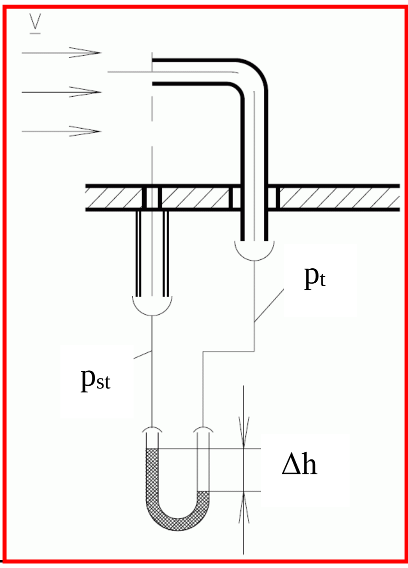


7. Explain velocity measurements using a Pitot probe! Draw a figure, too!

The Pitot probe is a tube turned against the flow using which the pressure of the stopped flow (total or stagnation pressure) can be measured. If the streamlines are straight and parallel to each other the static pressure can be measured through a wall bore. The difference of these is the dynamic pressure, that gives the velocity:

$$p_d = p_t - p_{st};$$

$$v = \sqrt{\frac{2 \cdot p_d}{\rho}} = \sqrt{\frac{2 \cdot \Delta p}{\rho}}$$

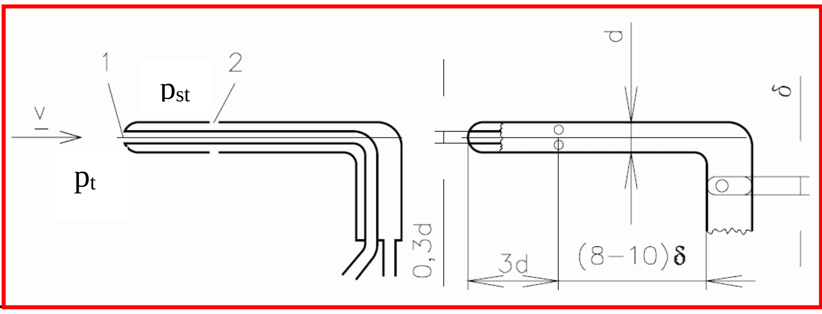


8. Explain velocity measurements using a Pitot-static or Prandtl probe! Draw a figure, too!

The Pitot-static or Prandtl probe consist of two, concentric tubes. The inner one is a Pitot probe that measures total pressure, while the outer one is connected to wall pressure bores that measure static pressure. Connecting both outlets to the same manometer gives the difference between the two pressures, the dynamic pressure. Velocity is calculated based on the dynamic pressure:

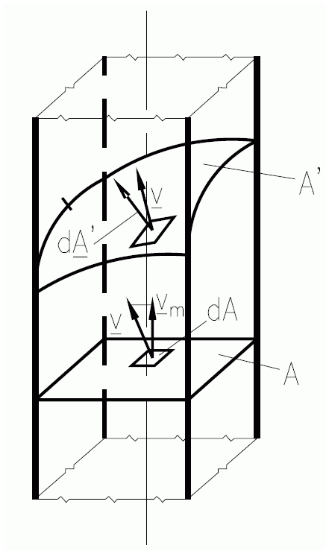
$$p_d = p_t - p_{st};$$

$$v = \sqrt{\frac{2 \cdot p_d}{\rho}} = \sqrt{\frac{2 \cdot \Delta p}{\rho}}$$



9. Explain volume flow rate measurement based on velocity measurements in case of pipes with circular and rectangular cross sections!

In both cases the total volume flow rate is given as the sum of partial volume flow rates. The local flow velocity is measured using a Pitot-static (Prandtl) probe. Measurements can only be performed while the operation state of the system does not change, since measurements have to be made one after the other in several points.

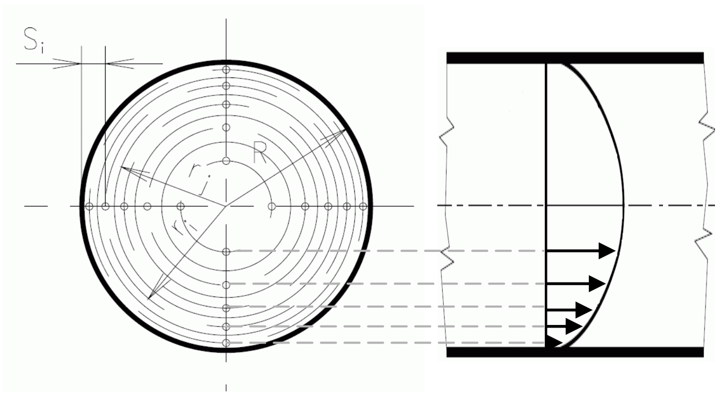


Rectangular cross section:
In this case the partial cross section pieces are of equal size, geometrically similar to the whole cross section and the velocity component perpendicular to the cross section has to be considered. Index m indicates the perpendicular component.

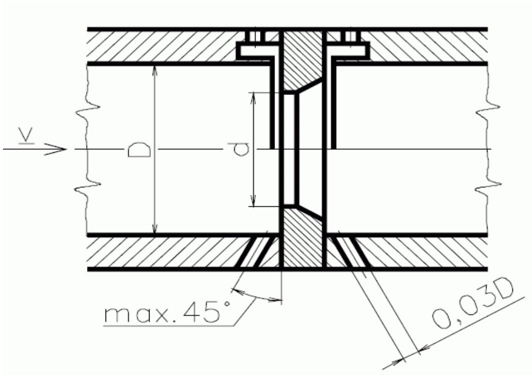
$$q_v = \int_A \underline{v} d\underline{A} = \sum_{i=1}^{4} q_{v,i} \approx \sum_{i=1}^{n} v_{m,i} \Delta A_i =$$

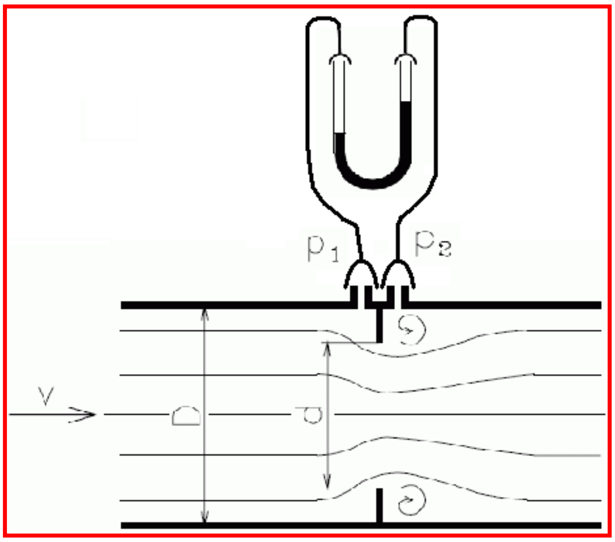
$$\frac{v_{m,1}A_1 + v_{m,2}A_2 + v_{m,3}A_3 + v_{m,4}A_4}{4} = A\bar{v}$$

Circular cross section:
Using the **10 point method**, the velocity profile is assumed to be parabolic and the measurement points are taken so that the **ring areas** defined by them **are equal** to each other.
In case of **turbulent** velocity profiles, the **6 point method** gives more accurate results.



10. Draw a sketch of volume flow rate measurements using an orifice! Show the orifice, the pressure bore locations and the connected manometer! Indicate the lower and higher pressures!





$p_1 > p_2$

11. Give the formula for the calculation of volume flow rate measured by an orifice! Give the names and units of the variables! Explain the meaning of the contraction ratio (α) and the expansion number (ε)!

$$\boxed{q_v = \alpha\varepsilon \frac{d^2\pi}{4}\sqrt{\frac{2\Delta p}{\rho}}}$$

q_v: volume flow rate, $[q_v] = m^3/s$
α: contraction ratio, $[\alpha]$ = -
ε: expansion number, $[\varepsilon]$ = -
d: diameter of the smallest cross section, [d] = m
Δp: pressure drop measured between the pressure outlets, $[\Delta p]$ = Pa
ρ: fluid density, $[\rho] = kg / m^3$

The contraction ratio is chosen based on standards depending on Reynolds number and the ratio between the smallest diameter to the pipe diameter. It's value for an inlet orifice is roughly 0.6.
The expansion number is taken to be unity as long as the pressure drop is small (below 5000 Pa).

12. Compare volume flow rate measurement methods based on velocity measurements to flow contraction methods! List their advantages and disadvantages!

ASPECT	CONTRACTION	VELOCITY-BASED
1/ Intrusiveness	“ – ” Considerable losses $\Rightarrow$ the operating state may be modified $\Leftrightarrow$ to be included already in the system design state	“ + ” Negligible intrusiveness (wall bores only)
2/ Following temporal changes in the operational state	“ + ” Follows unsteady flow rate continuously	“ – ” Does not follow (long-lasting surface integration) ($\Leftrightarrow$ correction..?)
3/ Requirements	“ – ” Strict (manufacturing, installation, system is to be halted…)	“ + ” Moderate (no requirements, only recommendations, the system may run continuously…)
4/ Expenses	“ – ” High (manufacturing, installation, operation: losses to be covered)	“ + ” Moderate
5/ Accuracy	“ + ” High (limited uncertainty, guaranteed by the standard) <u>Legally defensible!</u>	“ – ” Moderate (limits of uncertainty are not guaranteed) Legally <u>assailable</u>!

13. Define absolute and relative errors! How can you determine the relative error of a quantity that was calculated from several measured values?

In engineering practice the measured quantities are only know together with a measurement uncertainty. In order to find out how reliable the results are, error calculation has to be performed. Let X denote the measured quantity, and δX the measurement uncertainty of that. The correct way of providing measurement results is the following:

$$\boxed{X \pm \delta X}$$

where δX is the absolute error of quantity X,

$$\boxed{\frac{\delta X}{X}}$$

while this ratio is the relative error (usually given in a percentage form).

In most of the cases the error results from reading from the instruments. The reading error is approximately the finest scale interval. For example, in case of a U-tube manometer the level height is read from a scale interval of 1 mm, therefor the error in the level height is 1 mm. If a result is calculated from two quantities, the errors propagate. The relative error of the resulting quantity calculated from several, independently measured values is given by the following formula:

$$\boxed{\frac{\delta R}{R} = \sqrt{\sum_{i=1}^{n} \left(\frac{\delta X_i}{R} \cdot \frac{\partial R}{\partial X_i} \right)^2}}$$

where:

R is the calculated quantity,

X_i is one of the *n* measured quantities,

δX_i - is the absolute error of measuring X_i, and

δR is the absolute error of the calculated quantity.