Budapest University of Technology and Economics
Faculty of Mechanical Engineering

2002

DEPARTMENT OF FLUID MECHANICS

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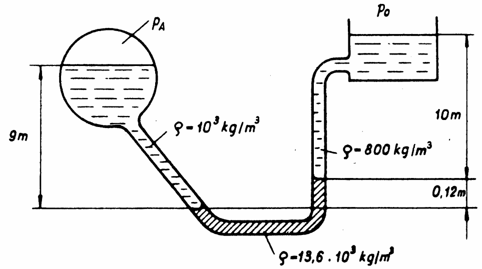
Selected Problems in Fluid Mechanics

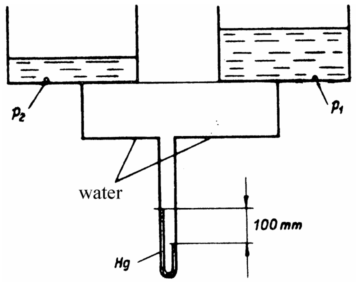
1 Hydrostatics

For all relevant problems $R = 287\ \mathrm{J/kg\,K}$, $g = 9.81\ \mathrm{N/kg}$

1/1 $p_A - p_0 = ?\ [\mathrm{Pa}]$



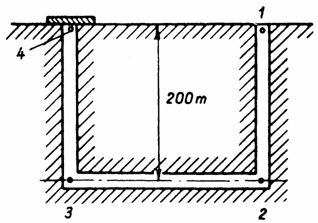
1/2 $p_1 - p_2 = ?\ [\mathrm{Pa}]$



1/3 Section 1-2: $\rho_{12} = 1.3\ \mathrm{kg/m^3}$

Section 3-4: $\rho_{34} = 1.1\ \mathrm{kg/m^3}$

$p_4 - p_1 = ?\ [\mathrm{Pa}]$

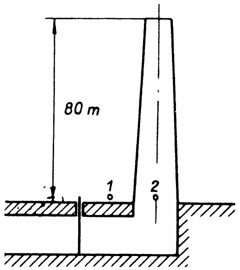


1/4 $p_0 \approx 10^5$ Pa (for the calculation of ρ)

Outside (air): $T_1 = 0\ ^\circ\mathrm{C}$

In chimney (smoke): $\left.\begin{matrix} p_2 \approx 760\ \mathrm{mmHg} \\ T_2 = 250\ ^\circ\mathrm{C} \end{matrix}\right\}$

$p_1 - p_2 = ?\ [\mathrm{Pa}]$

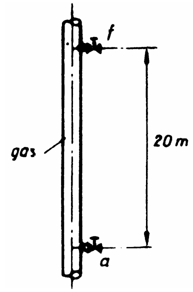


1/5 The figure shows a vertical section of a gas pipe. At the lower tap there is an overpressure of 500 Pa. How big is the overpressure at the upper tap?

There is no flow in the pipe.

$\rho_{air} = 1.2\ \mathrm{kg/m^3}$

$\rho_{gas} = 0.7\ \mathrm{kg/m^3}$



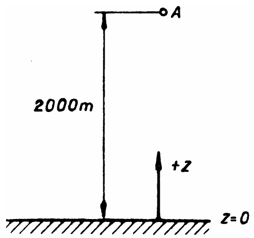
1/6

$z = 0 \left\{\begin{matrix} p_0 = 10^5\ \mathrm{N/m^2} \\ \rho_0 = 1.2\ \mathrm{kg/m^3} \end{matrix}\right\}$ air, $R = 288\ \mathrm{J/kgK}$

a.) $T_0 = ?\ [\mathrm{K}]$

b.) $p_A = ?\ [\mathrm{Pa}]$,

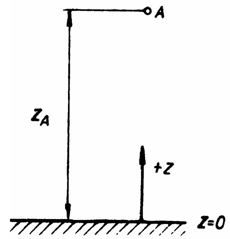
if the temperature is constant for $0 \le z < 2000\mathrm{m}$.



1/7 $p_A = 0.5 \cdot 10^5\ \mathrm{N/m^2}$

$z = 0 \left\{\begin{matrix} p_0 = 10^5\ \mathrm{N/m^2} \\ \rho_0 = 1.25\ \mathrm{kg/m^3} \end{matrix}\right\}$ air

$z_A = ?\ [\mathrm{m}]$ if the temperature is constant for $0 \le z < z_A$.

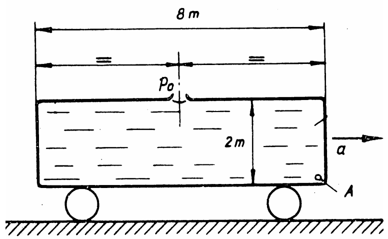


1/8 The vehicle is filled with oil.

$\rho_{oil} = 950\ \mathrm{kg/m^3}$

$a = 3\ \mathrm{m/s^2}$

$p_A - p_0 = ?\ [\mathrm{Pa}]$

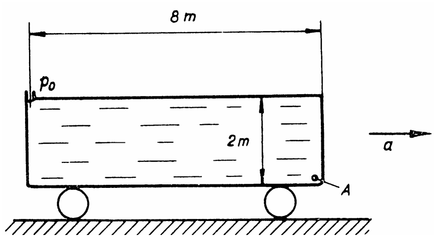


1/9 The vehicle is filled with oil.

$\rho_{oil} = 950\ kg/m^3$

$p_A - p_0 = 0\ Pa$

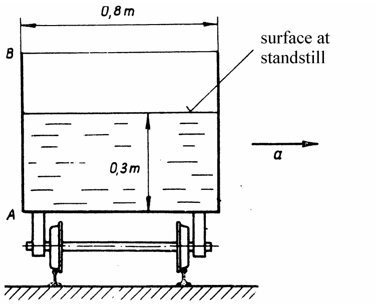
$a = ?\ [m/s^2]$



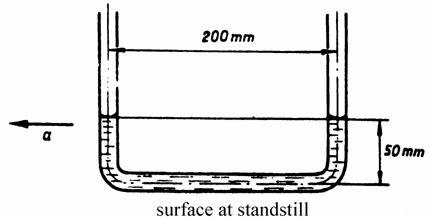
1/10 The tank wagon shown in the figure is taking a curve with a centripetal acceleration of $a = 3\ m/s^2$. The tank is filled with water.

a.) How high will climb the water surface on the A-B side?

b.) How big force will affect the A-B side, when the vehicle is 1.6 m long?



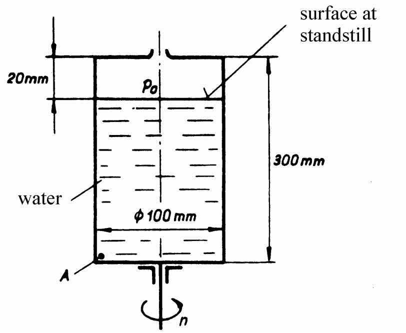
1/11 Where are the both surfaces of the liquid situated if the pipe accelerates to the left with an acceleration of $a = \frac{g}{2}$?



1/12 $n = 1000\ 1/min$

$\rho_{water} = 1000\ kg/m^3$

$p_A - p_0 = ?\ [Pa]$



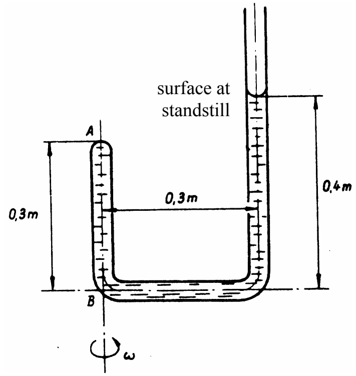
1/13 The pipe is filled with water.

$p_0 = 10^5\ Pa$

How high angular velocity is needed to

a.) reach $p_A = 0.8 \cdot 10^5\ Pa$?

b.) empty the A-B section and have pressure of $0.8 \cdot 10^5\ Pa$ in it?

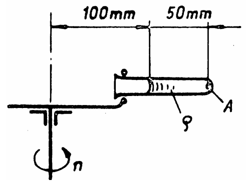


1/14 Effect of gravity is negligible.

$\rho = 800\ kg/m^3$

$n = 6000\ 1/min$

$p_A - p_0 = ?$



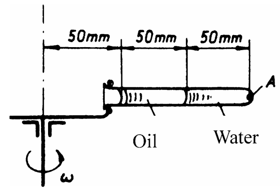
1/15 Effect of gravity is negligible.

$\omega = 100\ 1/s$

$\rho_{water} = 1000\ kg/m^3$

$\rho_{oil} = 800\ kg/m^3$

$p_A - p_0 = ?\ [Pa]$



1/16 What area does an ice-floe have, which can carry a person weighing 736 N? The thickness of the ice-floe is 10 cm and its density is 900 kg/m^3 ?

1/17 The rope is weightless.

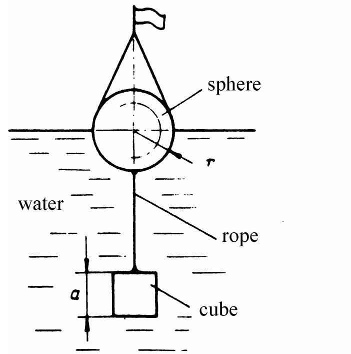
$\rho_{Cube} = 2300\ kg/m^3$

$\rho_{Water} = 1000\ kg/m^3$

$r_{Sphere} = 300\ mm$

$G_{Sphere} = 200N$

$a = ?\ [m/s^2]$

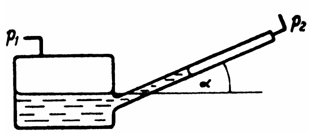


1/18 A balloon is filled with hot air of 60°C. Its diameter is 10 m. The environmental temperature is $0°C$. Pressure outside and inside the balloon is 10^5 Pa. The weight of the balloon material is can be neglected. Determine the buoyant force!

1/19 $p_1 - p_2 = 20\ N/m^2$

$\rho_{liquid} = 800\ kg/m^3$

$\alpha = ?\ [°]$ if an error of $\pm 1mm$ at the reading of the fluid column position causes $\pm 2\%$ relative error of $p_1 - p_2$.



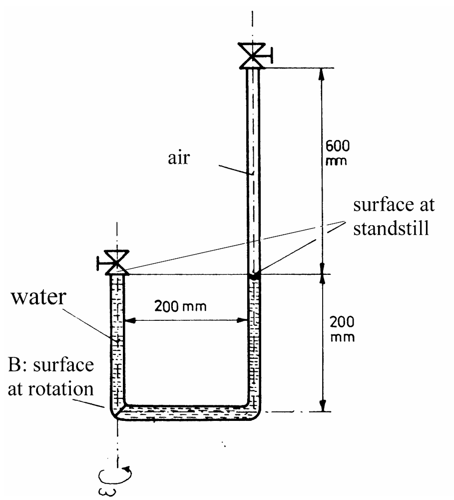
1/20 After having been filled the pipe both taps were closed. During the rotation the surface in the left pipe section sinks to the point B as shown in the figure.

$p_0 = 10^5\ Pa$

$p_{saturated\ steam} = 2 \cdot 10^4\ Pa$

$T = const$

$\omega = ?\ [1/s]$



2 Kinematics

2/1 Pressure changes are negligible.

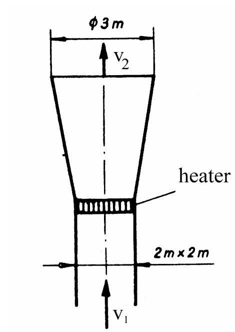
$q_v = 40\ m^3/s$

$t_1 = 15\ °C$

$t_2 = 80\ °C$

$v_1 = ?\ [m/s]$

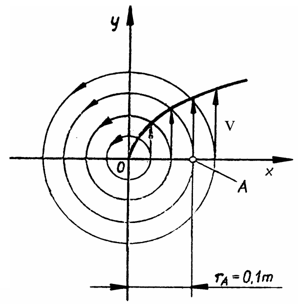
$v_2 = ?\ [m/s]$



2/2 Two dimensional flow:

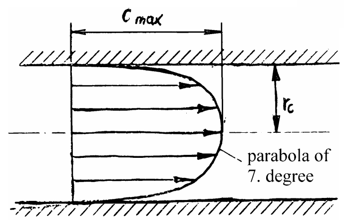
$|\underline{v}| = 10\sqrt{r}$

$[(\mathrm{rot}\ \underline{v})_z]_A = ?\ [1/s]$



2/3 Axisymmetric flow.

$\frac{v_{mean}}{v_{max}} = ?$

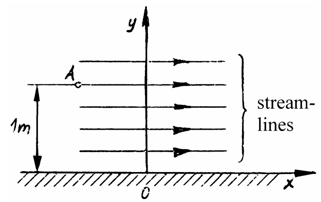


2/4 Unsteady, two dimensional flow.

$v_y = 0$

$v_x = 5yt^2$

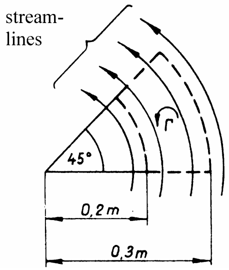
Calculate the local and convective acceleration in point 'A' at $t = 0.5\ s$.



2/5 Calculate the circulation along the dashed line.

$$v = \frac{2}{r^2}$$

$$\Gamma = ?\,\left[m^2/s\right]$$

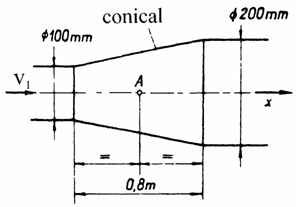


2/6

$v_1 = 20\ m/s$

$\rho = \text{const.}$

$\left[a_{konv}\right]_A = ?\,\left[m/s^2\right]$

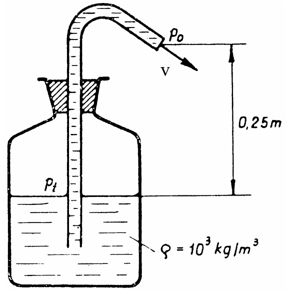


3 Bernoulli Equation

3/1 $p_t = 3 \cdot 10^5$ Pa

$p_0 = 10^5$ Pa

$v = ?\,[m/s]$

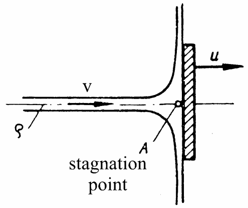


3/2 $v = 10\ m/s$

$u = 4\ m/s$

$\rho = 10^3\ kg/m^3$

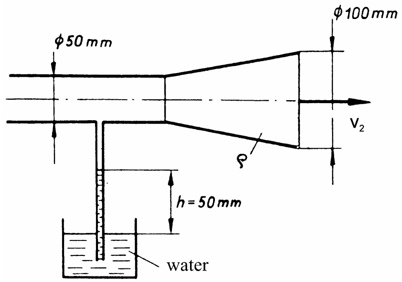
$p_A - p_0 = ?\,[Pa]$



3/3 Friction losses are negligible.

$\rho = 1.2\ kg/m^3$

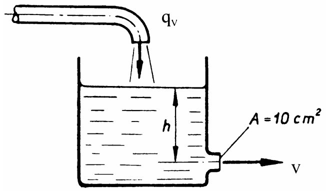
$v_2 = ?\,[m/s]$



3/4 Steady flow with

$q_V = 0.1\,m^3/min$.

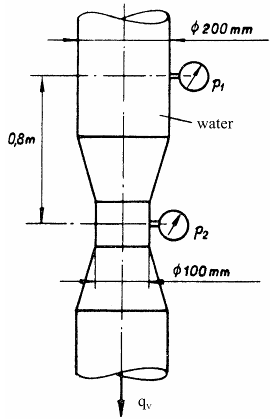
$h = ?\ [m]$



3/5 $p_1 = 1.6 \cdot 10^5$ Pa

$p_2 = 1.2 \cdot 10^5$ Pa

$q_V = ?\ [m^3/s]$

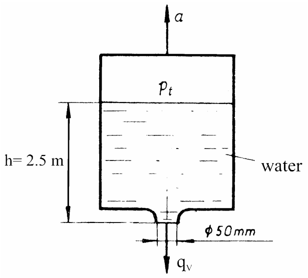


3/6 $a = 12\ m/s^2$

$p_0 = 10^5$ Pa

$p_t = 0.5 \cdot 10^5$ Pa

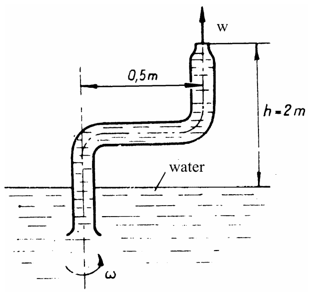
$q_V = ?\ [m^3/s]$



3/7 $\omega = 25\ 1/s$

$w = ?\ [m/s]$

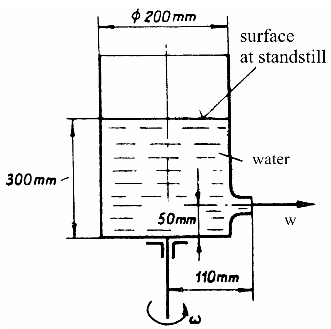
(w: relative velocity)



3/8 $w = 3\ m/s$

$\omega = ?\ [1/s]$

(w: relative velocity)

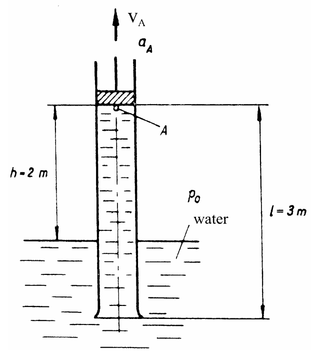


3/9 $p_0 = 10^5\ N/m^2$

$p_A = 0$

$v_A = 4\ m/s$

$a_A = ?\ [m/s^2]$



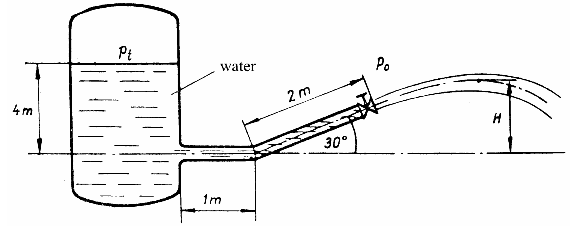
3/10 $p_0 = 10^5$ Pa

$p_1 = 0.9 \cdot 10^5$ Pa

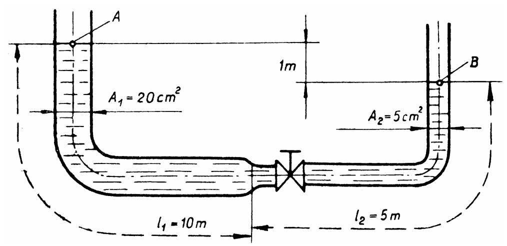
Friction losses are negligible.

a.) How big is the starting acceleration 'a' when opening the tap?

b.) $H = ?\ [m]$ in case of steady flow?



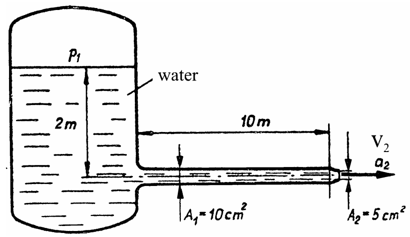
3/11 How big is the starting acceleration in point B when opening the tap?



3/12 How big is the starting acceleration at the end of the pipe?

$p_t = 2 \cdot 10^4\ N/m^2$ (overpressure)

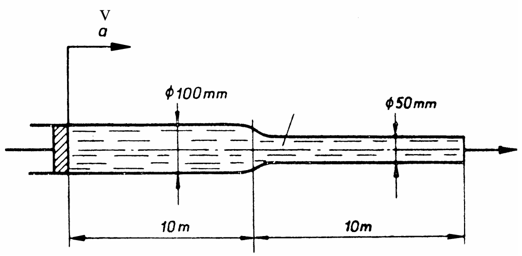
$v = 0$



3/13 $v = 1\ m/s$

$a = 1\ m/s^2$

Friction is negligible. How big force is needed to push the piston?



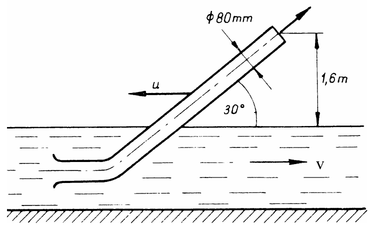
3/14 $u = 72\ km/h$

$v = 4\ m/s$

Friction is negligible.

a.) $q_V = ?\ \left[m^3/s\right]$

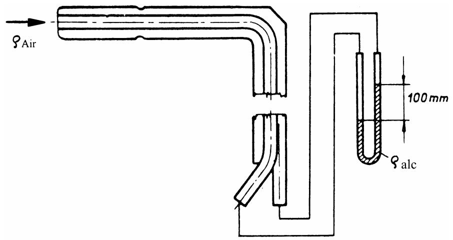
b.) How big power is needed to move the pipe?



3/15 $\rho_{alc} = 800\ kg/m^3$

$\rho_{air} = 1.2\ kg/m^3$

$v = ?\ \left[m/s\right]$



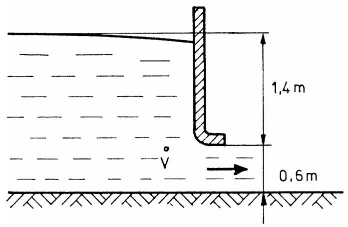
3/16 The inner diameter of an orifice flowmeter is $d = 200\ mm$. Flow coefficient $\alpha = 0.7$ Compressibility factor $\varepsilon = 1$. The measured difference pressure is $\Delta p = 600\ N/m^2$. $\rho = 1.3\ kg/m^3$.

$q_V = ?\ \left[m^3/s\right]$

3/17 Width of the flow is 1 m.

a.) Construct the velocity distribution diagram along the vertical line over the outlet.

b.) Calculate the flow rate q_V $\left[m^3/s\right]$!



3/18 Irrotational, horizontal, two-dimensional flow.

$r_1 = 0.5\ m$

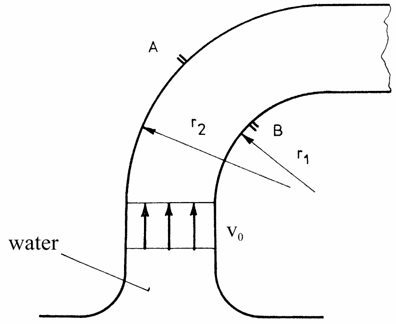
$r_2 = 0.8\ m$

$v_0 = 5\ m/s$

a.) What kind of velocity distribution has developed in the arc?

b.) $p_A - p_B = ?\ \left[Pa\right]$

c.) $\dfrac{p_A - p_B}{\rho \dfrac{v_0^2}{2}} = f\left(\dfrac{r_2}{r_1}\right)$? (Draw a diagram!)

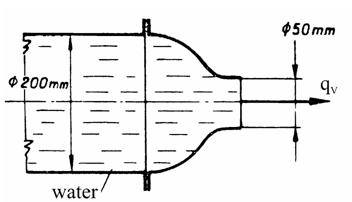


4 Integral Momentum Equation

4/1 Calculate the horizontal force acting on the conical part of the pipe!

$q_V = 3.5\ m^3/min$

Friction losses are negligible.



4/2 $v_1 = 30\ m/s$

$u = 13\ m/s$

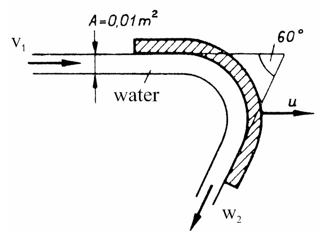
Friction losses are negligible.

a) $|\underline{v}_2| = ?\ [m/s]$

b) Calculate the angle of deviation $\beta\ [°]$ (angle between $\underline{v}_1$ and $\underline{v}_2$)!

c) Determine the force acting on the blade!

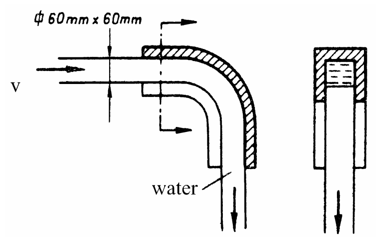
d) How is the kinetic energy of 1kg water changing, when passing the blade?



4/3 $v = 10\ m/s$

Friction and gravity are negligible.

Calculate the force acting on the arc!

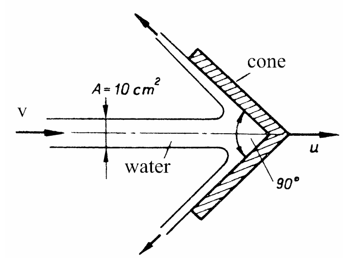


4/4 $v = 10\ m/s$

$u = 2\ m/s$

Friction and gravity are negligible.

Calculate the force acting on the moving conical body!

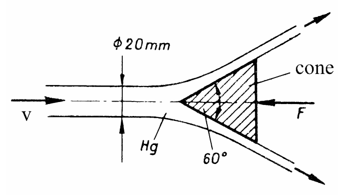


4/5 $v = 10\ m/s$

$\rho_{Hg} = 13600\ kg/m^3$

Friction and gravity are negligible.

Calculate the force acting on the cone!

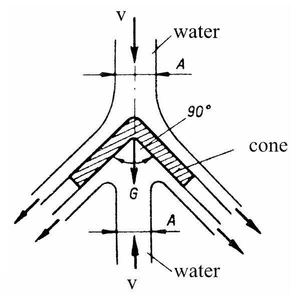


4/6 $A = 10^{-4}\ m^2$

$v = 10\ m/s$

Friction and gravity are negligible.

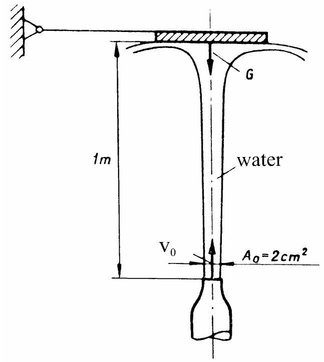
Determine the weight of body 'G' [N]!



4/7 $G = 1\ N$

$v_0 = ?\ [m/s]$

Friction is negligible.

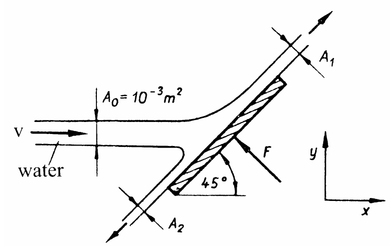


4/8 Two dimensional flow.

$v = 30\ m/s$

a) $F = ?\ [N]$

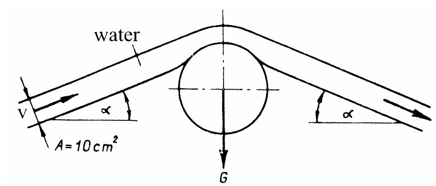
b) $A_1/A_2 = ?$



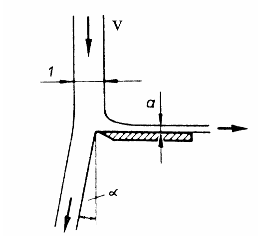
4/9 Two dimensional flow.

Friction and gravity are negligible.

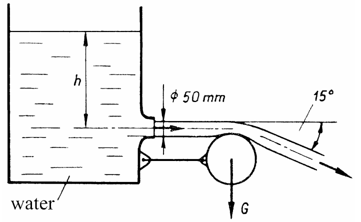
$\alpha = ?\ [°]$



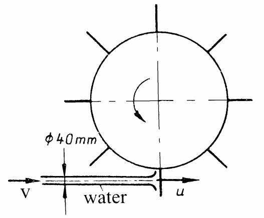
4/10 Two dimensional flow.
Friction losses are negligible.
$v = 10\ m/s$
$\alpha = 15°$
$G = ?\ [N]$



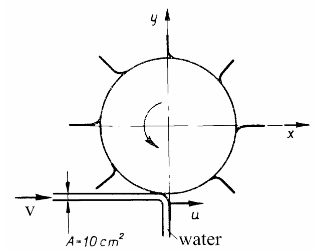
4/11 Friction losses are negligible.
The cylinder is balanced by the water jet.
$G = 10\ N$
$h = ?\ [m]$



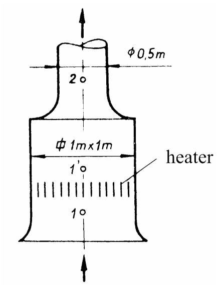
4/12 $v = 10\ m/s$
$u = 6\ m/s$
Friction is negligible.
Calculate the power transmitted by the water jet to the wheel!



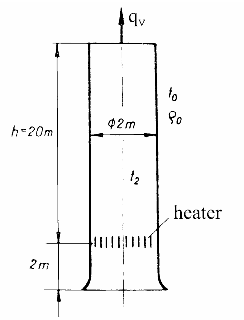
4/13 $v = 20\ m/s$
$u = 6\ m/s$
Friction is negligible.
Calculate the mean force acting on the wheel blades in the direction x and y!



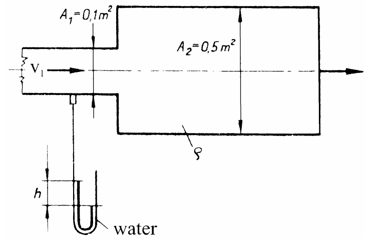
4/14 $v_1 = 2\ m/s$
$\rho_1 = 1.2\ kg/m^3$
$t_1 = 20°C$
$t_{1'} = t_2 = 300°C$
Friction, gravity and density changes of the air because of pressure changes are negligible.
$p_1 - p_2 = ?\ [Pa]$



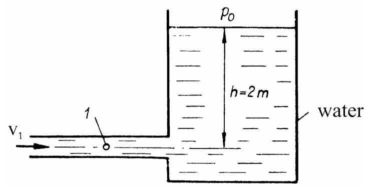
4/15 $v_1 = 2\ m/s$
$\rho_0 = 1.29\ kg/m^3$
$t_0 = 0°C$
$t_2 = 273°C$
Friction and density changes of the air because of pressure changes are negligible.
$q_V = ?\ [m^3/s]$



4/16 $v_1 = 20\ m/s$
$\rho = 1\ kg/m^3$
$h = ?\ [m]$

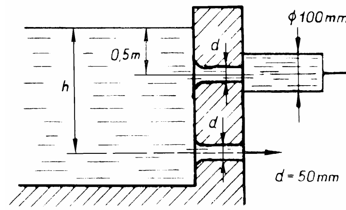


4/17 There is no friction loss in the pipe.
$p_1 - p_0 = ?\ [Pa]$

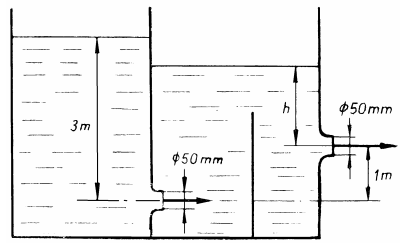


4/18 The flow rate through the lower and upper outlet is the same. The losses due to the rapid cross section change at the upper pipe must be considered.

$h = ?\ [m]$

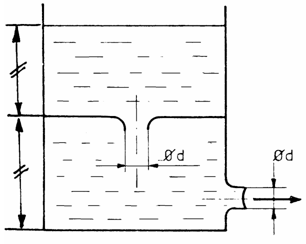


4/19 Steady flow.
$h = ?\ [m]$



4/20 Determine the quotient of the flow rates with and without horizontal plate!

$$\frac{q_V\big|^{\text{without plate}}}{q_V\big|^{\text{with plate}}} = ?$$



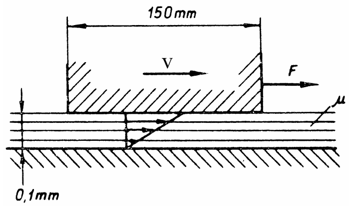
5 Hydraulics

5/1 The width of the gap is 100 mm (perpendicular to the paper plane).

$v = 0.5\ \mathrm{m/s}$

$\mu = 0.1\ \mathrm{kg/ms}$

$F = ?\ [\mathrm{N}]$



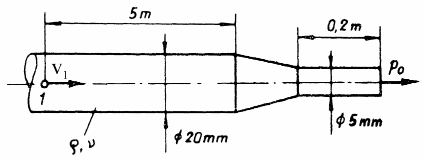
5/2 Friction loss in the confuser is negligible.

$v_1 = 0.5\ \mathrm{m/s}$

$\rho = 850\ \mathrm{kg/m^3}$

$\nu = 10^{-5}\ \mathrm{m^2/s}$

$p_1 - p_0 = ?\ [\mathrm{Pa}]$

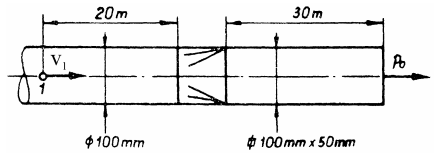


5/3 Friction loss of the transitional section is negligible.

$v_1 = 10\ \mathrm{m/s}$

$\rho = 1.2\ \mathrm{kg/m^3}$

$\nu = 14 \cdot 10^{-6}\ \mathrm{m^2/s}$



$p_1 - p_0 = ?\ [\mathrm{Pa}]$

5/4 How do the Reynolds number and the pressure loss of a straight, smooth pipe depend on diameter in case of laminar and turbulent flow, if the flow rate is constant?

5/5 How does a straight, smooth pipe's pressure loss depend on the flow rate in case of laminar and turbulent flow?

5/6 Oil flow rate of $q_V = 2 \cdot 10^{-4}\ \mathrm{m^3/s}$ has to be transported through a 10 m long straight pipe ($\rho = 800\ \mathrm{kg/m^3}$, $\nu = 10^{-4}\ \mathrm{m^2/s}$). The available pressure difference is not more than $2 \cdot 10^5$ Pa. Determine the diameter $D\ [\mathrm{mm}]$ of the pipe!

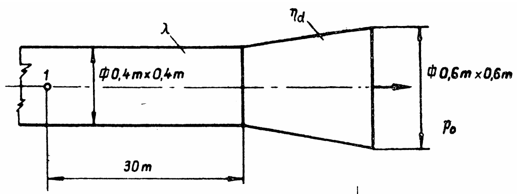
5/7 $q_V = 8000\ m^3/h$

$\rho = 1.2\ kg/m^3$

$\lambda = 0.025$

$\eta_D = 0.8$

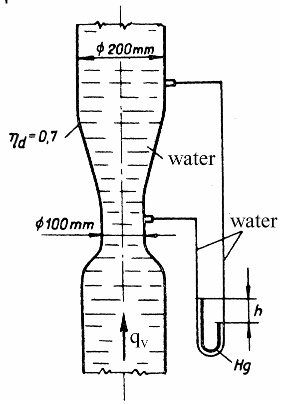
$p_1 - p_0 = ?\ [Pa]$



5/8 $q_V = 1200\ l/min$

$\rho_{Hg} = 13.6 \cdot 10^3\ kg/m^3$

$h = ?\ [m]$

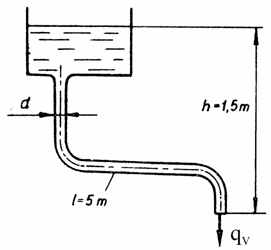


5/9 The figure shows a part of a lubrication equipment, which has to transport an oil flow rate of $q_V = 0.05 \cdot 10^{-3}\ m^3/s$. For the calculation of the friction loss, it can be considered that the pipe is straight.

$\rho_{oil} = 800\ kg/m^3$

$\nu_{oil} = 10^{-4}\ m^2/s$

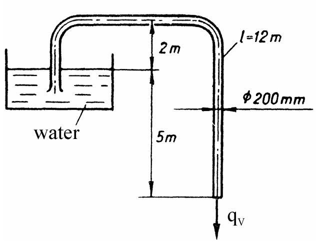
$d = ?\ [mm]$



5/10 The additional losses of the bends can be neglected. (It can be considered that the steel pipe is straight.)

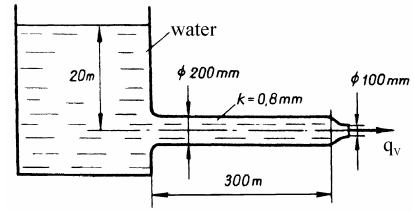
$\nu_{water} = 1.3 \cdot 10^{-6}\ m^2/s$

$q_V = ?\ [m^3/s]$



5/11 $\nu_{water} = 1.3 \cdot 10^{-6}\ m^2/s$

$q_V = ?\ [m^3/s]$

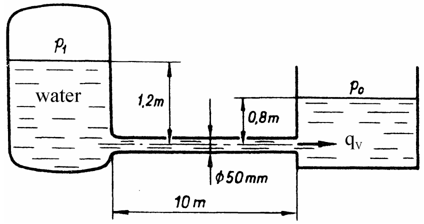


5/12 Hydraulically smooth pipe walls.

$\nu_{water} = 1.3 \cdot 10^{-6}\ m^2/s$

$q_V = 5\ l/s$

$p_1 - p_0 = ?\ [Pa]$

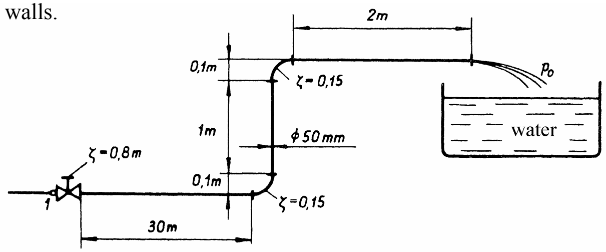


5/13 Hydraulically smooth pipe walls.

$\nu_{water} = 1.3 \cdot 10^{-6}\ m^2/s$

$q_V = 180\ l/min$

$p_1 - p_0 = ?\ [Pa]$



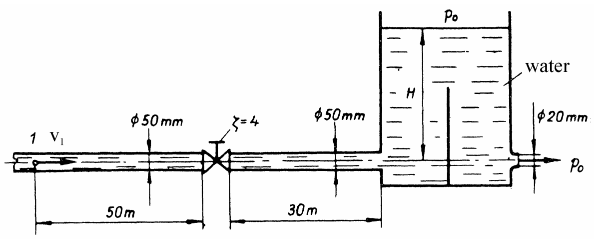
5/14 Steady flow, hydraulically smooth pipe.

$\nu_{water} = 1.3 \cdot 10^{-6}\ m^2/s$

$v_1 = 1\ m/s$

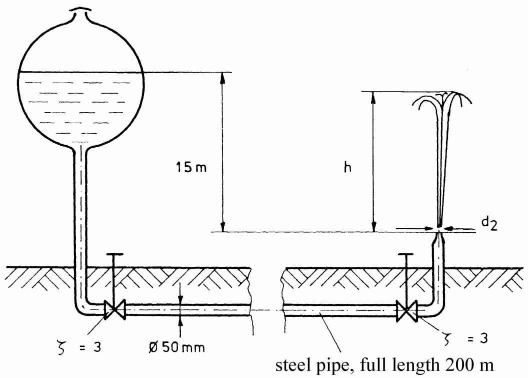
a) $H = ?\ [m]$

b) $p_1 - p_0 = ?\ [Pa]$

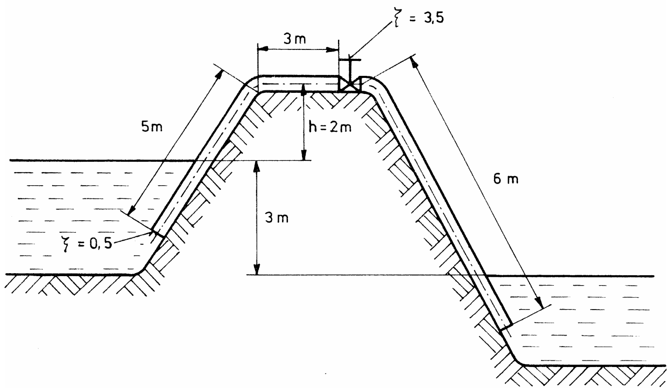


5/15 What power is needed to drive the shaft of a glide bearing with $2880\,1/\mathrm{min}$, when the shaft is $60\,\mathrm{mm}$ wide, 100mm long and the gap between bearing and shaft is 0.2 mm? ($\mu_{oil} = 0.01\,\mathrm{kg/ms}$) How is it possible to decrease this power?

5/16 a) Determine the confuser's output diameter d_2, when the water jet is 12 m high!
b) Calculate the flow rate $q_V\,[\mathrm{m^3/s}]$ through the pipe! Friction losses of the bends, the confuser and friction effects between the water jet and the air are negligible.

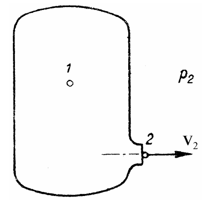


5/17 Water of $q_V = 18\,\mathrm{m^3/h}$ flow rate has to be transported by the equipment shown in the figure.
a) How wide pipe do we need to fulfill this task?
b) Determine the maximal dike height where the transport is possible? (theoretical answer)

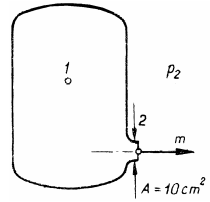


6 Compressible Flows

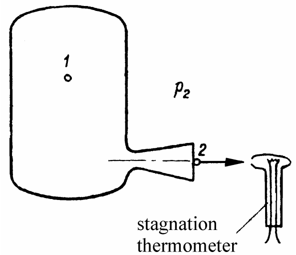
6/1 $p_1 = 1.5$ bar, $p_2 = 1$ bar
$T_1 = 300$ K
$c_p = 1000$ J/kg K
$\kappa = 1.4$
Isentropic change of state.
$v_2 = ?\ [\mathrm{m/s}]$



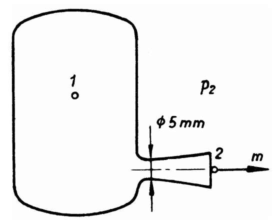
6/2 $p_1 = 1.3\cdot 10^5$ Pa, $p_2 = 10^5$ Pa
$T_1 = 273$ K
$R = 287$ J/kg K
$\kappa = 1.4$
Isentropic change of state.
$q_m = ?\ [\mathrm{kg/s}]$



6/3 $p_1 = 1.4$ bar, $p_2 = 1$ bar
$t_1 = 20$ °C
$\kappa = 1.4$
Isentropic change of state.
a) $t_{2\,static} = ?\,[°C]$
b) $t_{2\,total} = ?\,[°C]$
(temperature measured by the stagnation point thermometer)



6/4 $p_1 = 4$ bar, $p_2 = 1$ bar
$T_1 = 300$ K
$R = 287$ J/kg K
$\kappa = 1.4$
Isentropic change of state.
$q_m = ?\ [\mathrm{kg/s}]$



6/5 $p_1 = 4$ bar, $p_2 = 1$ bar

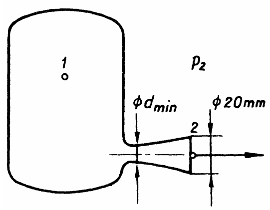
$t_1 = 70$ °C

$R = 287$ J / kg K

$\kappa = 1.4$

Isentropic change of state.

$d_{min} = ?$ [mm]



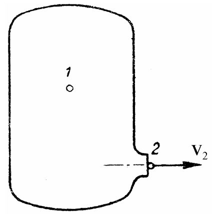
6/6 What kind of formula can be used to calculate v_2, if

a) $\frac{p_2}{p_1} = 0.99$

b) $\frac{p_2}{p_1} = 0.6$

c) $\frac{p_2}{p_1} = 0.4$

Isentropic change of state.



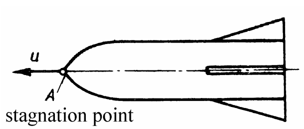
6/7 Air of temperature $t = -40$ °C flows at a velocity $v = 180$ m / s. $\kappa = 1.4$, $R = 287$ J / kg K. Calculate the Mach number (Ma) !

6/8 Carbon-dioxide of the temperature $t = 20$ °C flows at a Mach number of $Ma = 0.3$. $\kappa = 1.3$, $R = 189$ J / kg K.
Calculate the velocity of the flow! [m / s]

6/9 A rocket flies in air of $t = -23$ °C at a velocity of $u = 400$ m / s.

$c_p = 1000$ J / kgK

$t_A = ?$ [°C]

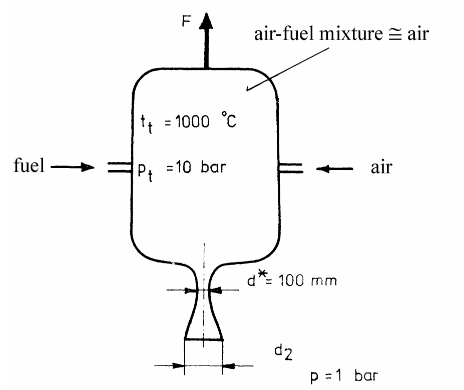


6/10 An aircraft flies in air of $t = 0$ °C at a velocity $u = 200$ m / s. The relative velocity w_2 in a definite point of the wing makes 250 m / s. $R = 287$ J / kg K, $\kappa = 1.4$. Calculate the Mach number in this point.

6/11 $R = 287$ J / kg K, $c_p = 1000$ J / kgK, $\kappa = 1.4$.

a) How wide should be the diameter d_2, if the outflow needs to be isentropic?

b) Calculate the thrust F [N] of the rocket engine!



RESULTS

1 Hydrostatics

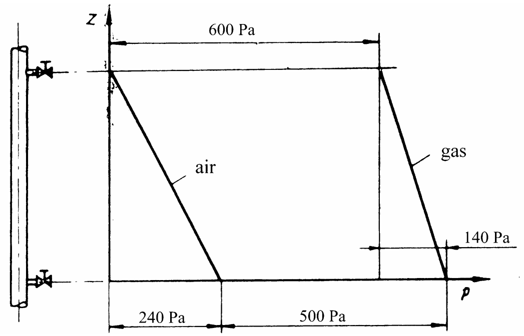
1/1 $p_A - p_0 = 6200\ \text{N/m}^2$

1/2 $p_1 - p_2 = 12360\ \text{N/m}^2$

1/3 $p_4 - p_1 = 392\ \text{N/m}^2$

1/4 $p_1 - p_2 = 486\ \text{N/m}^2$

1/5 The overpressure at the upper tap is 600 Pa.



1/6

a.) $T_0 = \dfrac{p_0}{\rho_0 R} = 290\ \text{K}$

b.) $\dfrac{dp}{dz} = -\rho g = -\dfrac{p}{p_0}\rho_0 g$

$$\int_{p_0}^{p_A} \frac{dp}{p} = -\frac{\rho_0 g}{p_0} z_A$$

$$\ln\frac{p_A}{p_0} = -\frac{\rho_0 g}{p_0} z_A$$

$$p_A = 0.788 \cdot 10^5\ \text{N/m}^2$$

1/7 $h = 5650\ \text{m}$

1/8 $p_A - p_0 = 7.23 \cdot 10^3\ \text{N/m}^2$

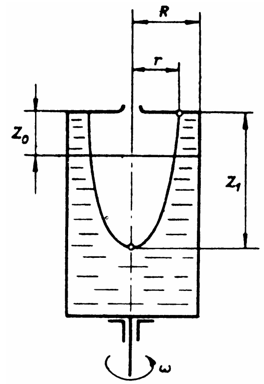
1/9 $a = 2.45\ \text{m/s}^2$

1/10 a) $h = 0.422\ \text{m}$

b) $F = 1400\ \text{N}$

1/11 The surface at the left side is situated at the left lower corner, the other surface in the right vertical section at a height of 100 mm.

1/12 Volumes are the same in standstill and rotation:



$$R^2 \pi z_0 = \frac{1}{2} r^2 \pi z_1$$

Points of equivalent potential:

$$g \cdot z_1 - \frac{r^2\omega^2}{2} = 0\ ;\ r^2 = \frac{2 g z_1}{\omega^2}$$

After substitution:

$$R^2 z_0 = \frac{1}{2}\frac{2 g z_1}{\omega^2} z_1 \quad z_1 = R\omega\sqrt{\frac{z_0}{g}} = 0.236\ \text{m}$$

$$p_A - p_0 = -\rho\left[g\,z_A - \frac{R^2\omega^2}{2}\right] = 14300\ \text{N/m}^2$$

1/13 After writing the equation

$$p = -\rho\left(gz - \frac{r^2\omega^2}{2}\right) + \text{const}\,.$$

for the both known points (surfaces in the left and the right section), the angular velocity can be calculated.

a.) $\omega = 21.4\ \ 1/\text{s}$

b.) $\omega = 24.3\ \ 1/\text{s}$

1/14 Equation $p = -\rho\left(gz - \dfrac{r^2\omega^2}{2}\right) + \text{const}$ written for the surface of the fluid:

$$\text{const.} = p_0 - \rho\frac{r_0^2\omega^2}{2}$$

$$p_A - p_0 = \rho\frac{\omega^2}{2}\left[r_A^2 - r_0^2\right] = 19.7 \cdot 10^5\ \text{N/m}^2$$

1/15 Apply the equation $p = -\rho\left(gz - \frac{r^2\omega^2}{2}\right) + \text{const}$ at first for the oil-filled part and then for the water filled part of the pipe. It can be written then:

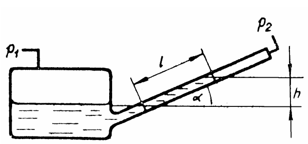
$$p_A - p_0 = \frac{\omega^2}{2}\left[\rho_{oil}\left(0.1^2 - 0.05^2\right) + \rho_{water}\left(0.15^2 - 0.1^2\right)\right] = 9.25\cdot 10^4\ \text{N/m}^2$$

1/16 $A = 7.5\ \text{m}^2$

1/17 $a = 0.3\ \text{m}$

1/18 $F = 1200\ \text{N}$

1/19 $h = \frac{20}{800\cdot 9.81} = 2.55\ \text{mm}$



$$l = \frac{\pm 1\text{mm}}{\pm 0.02} = 50\ \text{mm}$$

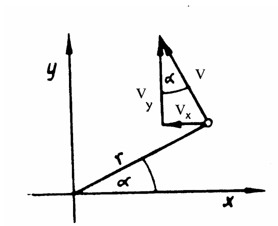
$$\sin\alpha = \frac{2.55}{50} = 0.051 \Rightarrow \alpha = 2.9°$$

1/20 $\omega = 81.8\ \text{1/s}$

2 Kinematics

2/1 $v_1 = 10\ \text{m/s}\,;\ v_2 = 6.9\ \text{m/s}$

2/2 Solution with Cartesian coordinates:



$$v_x = c(-\sin\alpha) = -v\frac{y}{r}\,;\ \ v_y = v\cdot\cos\alpha = v\frac{x}{r}$$

$$v_x = -10\sqrt{r}\,\frac{y}{r} = -10\frac{y}{\sqrt{r}} = -10\frac{y}{\sqrt[4]{x^2+y^2}}$$

$$v_y = 10\sqrt{r}\,\frac{x}{r} = 10\frac{x}{\sqrt{r}} = 10\frac{x}{\sqrt[4]{x^2+y^2}}$$

$$\frac{\partial v_y}{\partial x} = 10\frac{\sqrt[4]{x^2+y^2} - x\frac{1}{4}\left(x^2+y^2\right)^{-3/4}2x}{\sqrt{x^2+y^2}} \quad \text{at point A} : x, y = (0.1, 0) \Rightarrow \frac{\partial v_y}{\partial x} = 50\sqrt{0.1}$$

$$\frac{\partial v_y}{\partial x} = 10\frac{\sqrt[4]{x^2+y^2} - x\frac{1}{4}\left(x^2+y^2\right)^{-3/4}2x}{\sqrt{x^2+y^2}} \quad \text{at point A} \Rightarrow \frac{\partial v_x}{\partial y} = -100\sqrt{0.1}$$

$$\left[(\text{rot}\ \underline{v})_z\right]_A = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = (50+100)\sqrt{0.1} = 47.5\ \text{1/s}$$

Solution with polar coordinates:

$$\left[(\text{rot}\ \bar{c})_z\right]_A = \left[\frac{dc}{dr} + \frac{c}{r}\right]_A = 10\frac{1}{2\sqrt{r}} + \frac{10}{\sqrt{r}} = \frac{15}{\sqrt{r}} = \frac{15}{\sqrt{0.1}} = 47.5\ \text{1/s}$$

2/3

$$v = v_{max}\left[1 - \left(\frac{r}{r_0}\right)^7\right]$$

The cross section has to be divided into rings of elementary width 'dr'. Integrate the elementary flow rate through the rings as follows:

$$v_{mean} = \frac{1}{r_0^2\pi}\int_0^{r_0} 2r\pi v(r)\,dr = \int_0^1 2\frac{r}{r_0}v\left(\frac{r}{r_0}\right)\,d\left(\frac{r}{r_0}\right) = 2\int_0^1 \frac{r}{r_0}v_{max}\left[1 - \left(\frac{r}{r_0}\right)^7\right]d\left(\frac{r}{r_0}\right) =$$

$$\left[2\,v_{max}\frac{1}{2}\left(\frac{r}{r_0}\right)^2 - 2\,v_{max}\frac{1}{9}\left(\frac{r}{r_0}\right)^9\right]_0^1 = v_{max}\left(1 - \frac{2}{9}\right) = \frac{7}{9}v_{max} \Rightarrow \frac{v_{mean}}{v_{max}} = \frac{7}{9} = 0.778$$

In general:

$$v = v_{max}\left[1 - \left(\frac{r}{r_0}\right)^n\right] \Rightarrow \frac{v_{mean}}{v_{max}} = \frac{n}{n+2}$$

2/4 $\left[a_{local}\right]_{t=0.5}^{y=1} = 5\ \text{m/s}^2$

$a_{convective} = 0$

2/5 $\Gamma = \oint \underline{v}\,d\underline{s} = -2.61\ \text{m}^2/\text{s}$

2/6 $r_1^2\pi v_1 = r^2\pi v$

$$v = v_1 r_1^2\frac{1}{r^2}$$

$$\frac{\partial v}{\partial x}=\frac{\partial v}{\partial r}\frac{\partial r}{\partial x}=v_1 r_1{}^2\left[-\frac{2}{r^3}\right]\frac{\Delta r}{\Delta x}$$

$$a_{convective}=v\frac{\partial v}{\partial x}=-\frac{2v_1^2 r_1^4}{r^5}\frac{\Delta r}{\Delta x}$$

$$\left[a_{convective}\right]_A=-\frac{2\cdot 20^2 0.05^4}{0.075^5}\frac{0.05}{0.8}=-132\ m/s^2$$

3 Bernoulli Equation

3/1 $\frac{p_t}{\rho}=\frac{v^2}{2}+\frac{p_0}{\rho}+g\cdot h$

$v=19.8\ m/s$

3/2 $p_A-p_0=\frac{\rho}{2}(v-u)^2=1.8\cdot 10^4\ Pa$

3/3 $\rho_{water}\cdot g\cdot h=\frac{\rho}{2}v^2\left[\left(\frac{100}{50}\right)^4-1\right]\ \Rightarrow\ v=7.4\ m/s$

3/4

$$h=\frac{\left(\frac{q_V}{A}\right)^2}{2g}=0.141\ m$$

3/5 $q_V=0.793\ m^3/s$

3/6 $\frac{p_t}{\rho}+(g+a)\cdot h=\frac{p_0}{\rho}+\frac{v^2}{2}$

$q_v=0.00589\ m^3/s$

3/7 Observing in an absolute co-ordinate system, the flow is irrotational ($\mathrm{rot}\ \underline{v}=0$). In a co-ordinate system rotating with the pipe, $\mathrm{rot}\ \underline{w}=2\omega$, so the term $\int \underline{w}\times\mathrm{rot}\ \underline{w}\ d\underline{s}$ is equal to $\int 2\ \underline{w}\times\underline{\omega}\ d\underline{s}$, the Coriolis force term. ($\underline{w}$ – relative velocity) The Bernoulli equation can be written after simplifying the terms above:

$$\frac{(-r_1\omega)^2}{2}-\frac{r_1{}^2\omega^2}{2}=\frac{v_2^2}{2}+g\cdot h-\frac{r_2{}^2\omega^2}{2}$$

Point 1 is situated on the water surface on an arbitrary radius r_1, point 2 at the upper end of the pipe.

$v_2=10.8\ m/s$

3/8 $\omega=24\ \ 1/s$

3/9 $\frac{p_0}{\rho}=\frac{v_A{}^2}{2}+g\cdot h+\int_0^A\frac{\partial v}{\partial t}d\underline{s}$

$$\int_0^A\frac{\partial v}{\partial t}d\underline{s}=a_A\cdot l=a_A\cdot 3m$$

$a_A=24.1\ m/s^2$

3/10 a.) $[a]_{t=0}=6.55\ m/s$

b.) $H=1.52\ m$

3/11 $\int_A^B\frac{\partial v}{\partial t}d\underline{s}=a_B\left[10\frac{5}{20}+5\right]=7.5a_B$

$[a_B]_{t=0}=1.31\ m/s^2$

3/12 $[a_2]_{t=0}=7.94\ m/s^2$

3/13 $F=451\ N$

3/14 a) The Bernoulli-Equation has to be written between the surface point (1) and the pipe's outlet point (2), in a co-ordinate system moving with the pipe. It means that $v_1=24\ m/s$.

From the Bernoulli-equation:

$v_2=23.4\ m/s\ \Rightarrow\ q_V=0.116\ m^3/s$

b) the power is necessary to lift the water and to increase its kinetic energy. The change of the kinetic energy must be calculated with the absolute velocity 'v'.

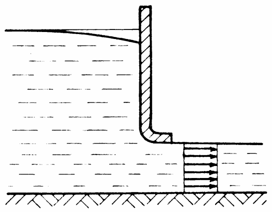
$$P=\rho\cdot q_V\left[g\cdot h+\frac{v_2{}^2-v_1{}^2}{2}\right]=8.85\ kW.$$

3/15 $v=\sqrt{\frac{2\cdot\Delta p}{\rho_{air}}}=36\ m/s$

3/16 $q_V = \alpha \cdot \varepsilon \dfrac{d^2\pi}{4}\sqrt{\dfrac{2\Delta p}{\rho}} = 0.67\ m/s$

3/17 Because the stream lines leaving the outlet are straight and parallel, there is only a hydrostatic pressure variation along the vertical axis. It follows that the outlet velocity is constant.

$q_V = 3.15\ m^3/s$.



3/18 a) in the arc $v = \dfrac{K}{r}$, because $\mathrm{rot}\ \underline{v} = 0$.

b) $v_{mean} = \dfrac{1}{r_2 - r_1}\int_{r_1}^{r_2}\dfrac{K}{r}dr = \dfrac{K}{r_2 - r_1}\ln\dfrac{r_2}{r_1}$ Because of continuity: $v_{mean} = v_0$

$$\Rightarrow K = \frac{v_{mean}(r_2 - r_1)}{\ln\dfrac{r_2}{r_1}} = 3.2$$

$$\Rightarrow v_A = \frac{K}{r_2} = 4\ m/s,\quad v_B = \frac{K}{r_1} = 6.4\ m/s$$

From the Bernoulli-equation:

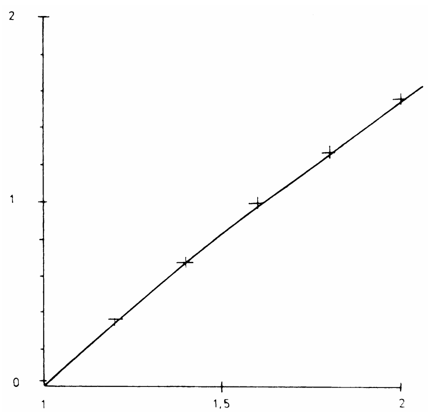
$$p_A - p_B = \frac{\rho}{2}\left(v_B{}^2 - v_A{}^2\right) = 1.25\cdot 10^4\ Pa$$

c.)

$$\frac{p_A - p_B}{\dfrac{\rho}{2}v_0{}^2} = \left(\frac{v_B}{v_0}\right)^2 - \left(\frac{v_A}{v_0}\right)^2 = \ldots$$

$$\ldots = \frac{(n-1)^3}{\ln^2 n}\,\frac{n+1}{n^2}$$

with $n = \dfrac{r_2}{r_1}$



4 Integral Momentum Equation

4/1 $F_x = 12100\ N$

4/2 After writing the Bernoulli equation for points situated upstream and downstream the blade we get the result:

$|\underline{v}_2| = |\underline{v}_1|$

4/3 $F = 510\ N$, direction 45° from the horizontal plane ('Northeast')

4/4 $F = 109\ N$

4/5 $F = 57\ N$

4/6 $G = 14\ N$

4/7 The integral momentum equation written for a control surface including only the plate and the upper end of the jet:

$$G = \rho \cdot A \cdot v^2 = \rho \cdot A_0 \cdot v_0 \cdot v$$

with v, the speed at the lower surface of the control surface.

According to the Bernoulli equation:

$$v = \sqrt{v_0{}^2 - 2\cdot g \cdot h}$$

$$v_0 = 4.55\ m/s$$

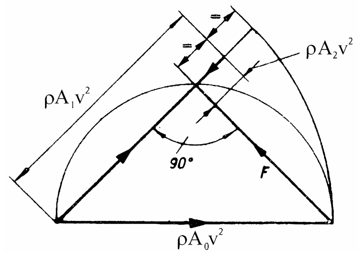
4/8 Write the integral momentum equation for both directions x and y:

a) $F = 636\ N$

b) $A_1 / A_2 = 5.8$

Solution with constructing the momentum rate vectors:

(It has to be considered that $\left|\rho \cdot A_0 \cdot v^2\right| = \left|\rho \cdot A_1 \cdot v^2\right| + \left|\rho \cdot A_2 \cdot v^2\right|$)



4/9 $\alpha = \arcsin \dfrac{a}{1-a}$

4/10 $G = 52\ \text{N}$

4/11 $h = 1\ \text{m}$

4/12 $P = u \cdot \rho \cdot A \cdot v \cdot (v - u) = 302\ \text{W}$

4/13 $F_x = F_y = 280\ \text{N}$

4/14 $p_1 - p_{1'} = \rho_1 \cdot v_1 (v_{1'} - v_1)$

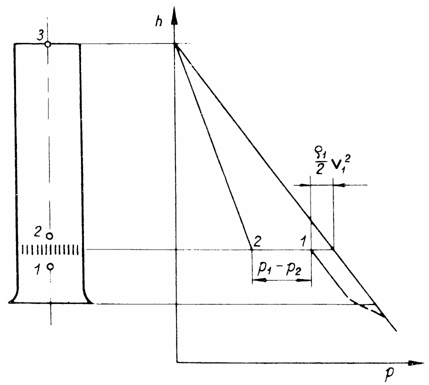
$p_{1'} - p_2 = \dfrac{\rho_2}{2}(v_2{}^2 - v_{1'}{}^2)$

$p_1 - p_2 = 123\ \text{Pa}$

4/15 $p_1 - p_2 = (\rho_1 - \rho_2) \cdot g \cdot h - \dfrac{\rho_1}{2} v_1{}^2$

$p_1 - p_2 = \rho_1 \cdot v_1 (v_2 - v_1)$

$q_V = 51\ \text{m}^3/\text{s}$



4/16 $A_2 (p_1 - p_2) = \rho \cdot A_2 v_2 (v_2 - v_1)$

$h = 6.5\ \text{mm}$

4/17 The Bernoulli-equation between point 1 and 2 (point 2 is situated at the outflow end of the pipe):

$\dfrac{v_1{}^2}{2} + \dfrac{p_1}{\rho} = \dfrac{v_2{}^2}{2} + \dfrac{p_0 + \rho \cdot g \cdot h}{\rho}$ because the area of cross section of the pipe is constant,

ρ=const, $v_2 = v_1$

An other solution can be the Bernoulli equation between point 1 and 3 (point 3 is situated on the water surface):

$\dfrac{v_1{}^2}{2} + \dfrac{p_1}{\rho} = \dfrac{v_2^2}{2} + \dfrac{p_0}{\rho} + g \cdot h + \dfrac{\Delta p_{B-C}}{\rho}$ where $\Delta p_{B-C} = \dfrac{\rho}{2}\left(v_2 - v_3\right)^2$ (Borda-Carnot-loss)

$v_2 = v_1$ and $v_3 = 0$.

4/18 $h = 0.8\ \text{m}$

4/19 $h = 1\ \text{m}$

4/20 $\dfrac{q_V\big|^{\text{without plate}}}{q_V\big|^{\text{with plate}}} = \sqrt{2}$

5 Hydraulics

5/1 $F = A \cdot \mu \cdot \dfrac{dv}{dy} = 7.5\ \text{N}$

5/2 $p_1 - p_0 = 72400\ \text{Pa}$

5/3 $p_1 - p_0 = 1500\ \text{Pa}$

5/4 $\text{Re} = \dfrac{q_v \cdot d}{\dfrac{d^2\pi}{4}\nu} = \dfrac{\text{const}}{d}$

$\Delta p_{\text{lam}} = \dfrac{\rho}{2} \dfrac{q_v{}^2}{\dfrac{d^4\pi^2}{16}} \dfrac{L}{d} \dfrac{64}{\dfrac{\text{const}}{d}} = \dfrac{\text{const}}{d^4}$

$\Delta p_{\text{turb}} = \dfrac{\rho}{2} \dfrac{q_v{}^2}{\dfrac{d^4\pi^2}{16}} \dfrac{L}{d} \dfrac{0.316}{\sqrt[4]{\dfrac{\text{const}}{d}}} \approx \dfrac{\text{const}}{d^5}$

5/5 $\Delta p_{lam} = \frac{\rho}{2}\frac{q_v^2}{A^2}\frac{L}{d}\frac{64}{\frac{q_v d}{A \cdot \nu}} = const \cdot q_V$

$\Delta p_{turb} = \frac{\rho}{2}\frac{q_v^2}{A^2}\frac{L}{d}\frac{0.316}{\sqrt[4]{\frac{q_v d}{A \cdot \nu}}} = const \cdot q_V^{1.75}$

5/6 Considered that the flow will be laminar and using the formula $\lambda = 64/Re$, we get $d = 13.4$ mm.

The Reynolds number is 189 which is less than 2300, so the flow is laminar.

5/7 $p_1 - p_0 = 143$ Pa

5/8 $h = 17$ mm

5/9 $g \cdot h = \frac{v^2}{2}\left(1 + \frac{L}{d}\lambda\right)$

Considering laminar flow, the result will be $d = 19.3$ mm.

$Re = 33 < 2300$, so the flow is really laminar.

5/10 $q_v = 0.23\ m^3/s$

5/11 $q_v = 0.0817\ m^3/s$

5/12 $p_1 - p_0 = 10900$ Pa

5/13 $p_1 - p_0 = 28500$ Pa

5/14 a) $H = 2$ m

b) $p_1 - p_0 = 40000$ Pa

5/15 $P = 77$ W

The power can be decreased by sinking the oil viscosity and by increasing the gap.

5/16 The resultant height loss is $h_{res} = 15\ m - 12\ m = 3\ m$.

$g \cdot h_{res} = \frac{v^2}{2}\left(\frac{L}{d}\lambda + 2\zeta\right)$

Starting with $\lambda = 0.02$, $v_{pipe} = \sqrt{\frac{3m \cdot 2 \cdot 9.81 m/s^2}{\frac{200m}{0.05m}0.02 + 6}} = 0.827\ m/s$

$Re = \frac{0.827 \cdot .05}{1.3 \cdot 10^{-6}} = 3.2 \cdot 10^4 \Rightarrow \lambda = 0.024$

After the next iteration step, $v_{pipe} = 0.755\ m/s$, and the iteration can be finished.

To reach $h = 12$ m, the necessary velocity at the confuser's outlet must be:

$v_2 = \sqrt{2 \cdot g \cdot h} = 15.3\ m/s$

$d_2 = \sqrt{\frac{0.755\ m/s}{15.3\ m/s}} \cdot 50\ mm = 11\ mm$

$q_v = 1.47 \cdot 10^{-3}\ m^3/s$

5/17 a) At first the velocity without friction loss can be calculated: $v_{ideal} = \sqrt{2 \cdot g \cdot 3m} = 7.7 m/s$,

and $A = \frac{\frac{18}{3600} m^3/s}{7.7\ m/s} = 6.5 \cdot 10^{-4} m^2$

So the pipe diameter is in this case 29 mm. Because of friction losses, we need a pipe of larger diameter. We start the iteration with $\lambda = 0.02$ and $d = 50$ mm:

$v = \sqrt{\frac{3m \cdot 2 \cdot 9.81 m/s^2}{\frac{14m}{0.05m}0.02 + 4 + 1}} = 2.36\ m/s \Rightarrow A = 21.2 \cdot 10^{-4} m^2 \Rightarrow d = 52\ mm$

$Re = \frac{2.36 \cdot 0.052}{1.3 \cdot 10^{-6}} = 9.45 \cdot 10^4 \Rightarrow \lambda = 0.018$

(At this Reynolds number we consider that the pipe is hydraulically smooth)

In the next iteration step with $\lambda = 0.018$ and $d = 52$ mm we get the new diameter of 51.2 mm. The iteration can be finished.

b) If the dike is higher, the pressure in the pipe can reach the pressure of saturated steam. In this case, the water column is going to break. The lowest pressure appears after the valve, at the upper right point of the dike. From the equation

$p_{min} = p_0 - \rho \cdot g \cdot h_{max} - \frac{\rho}{2}v^2\left[1 + \frac{L_1 + L_2}{d}\lambda + \zeta\right]$

h_{max} can be calculated.

6 Compressible Flows

6/1 $v_2 = 260\ \mathrm{m/s}$

6/2 $q_m = A_2 \rho_2 v_2 = 10^{-3}\,\mathrm{m}^2 \cdot 1.37\ \mathrm{kg/m^3} \cdot 200 = 0.274\ \mathrm{kg/s}$

6/3 a) $t_{2\,\mathrm{static}} = -42°\mathrm{C}$

b) $t_{2\,\mathrm{total}} = +20°\mathrm{C}$

6/4 $\dfrac{T^*}{T_1} = \dfrac{2}{\kappa+1} = 0.833$

$a_1 = \sqrt{\kappa \cdot R \cdot T_1} = 346\ \mathrm{m/s}$

$a^* = \sqrt{\dfrac{T^*}{T_1}}\, a_1 = 316\ \mathrm{m/s} = v^*$

$\rho^* = \left(\dfrac{T^*}{T_1}\right)^{\frac{1}{\kappa-1}} \rho_1 = 2.9\ \mathrm{kg/m^3}$

$q_m = v^* \cdot \rho^* \cdot A^* = 0.018\ \mathrm{kg/s}$

6/5 $q_m = A_2 \cdot \rho_2 \cdot v_2 = 0.25\ \mathrm{kg/s}$

$A^* = \dfrac{q_m}{v^* \cdot \rho^*} = 2.34 \cdot 10^{-4}\,\mathrm{m}^2$

$d_{min} = d^* = 17.3\ \mathrm{mm}$

6/6 a) $v_2 = \sqrt{\dfrac{2}{\rho}(p_1 - p_2)}$

b) $v_2 = \sqrt{\dfrac{2\kappa}{\kappa-1}\dfrac{p_1}{\rho_1}\left[1 - \left(\dfrac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}}\right]}$

c) $v_2 = \sqrt{\dfrac{2\kappa}{\kappa-1}\dfrac{p_1}{\rho_1}\left[1 - \dfrac{2}{\kappa+1}\right]}$

6/7 $Ma = 0.59$

6/8 $v = 80\ \mathrm{m/s}$

6/9 $t_A = 56°\mathrm{C}$

6/10 $T_2 = 262\ \mathrm{K},\ Ma_2 = 0.77$

6/11 a) $d = 138\ \mathrm{mm}$

b) $F = \rho_2 \cdot A_2 \cdot v_2^{\,2} = 9.8 \cdot 10^3\,\mathrm{N}$