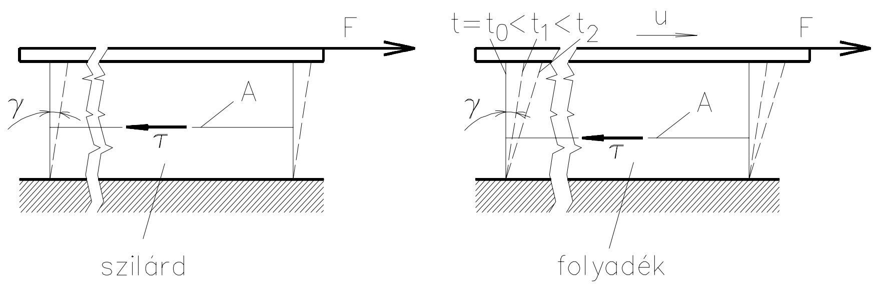
FLUID MECHANICS

1. Division of Fluid Mechanics

	Hydrostatics	Aerostatics	Hydrodynamics	Gasdynamics
v velocity				
p pressure				
ρ density				

2. Properties of fluids

Comparison of solid substances and fluids



solid fluid

$\tau = F/A$ [Pa] shear stress

Solid	γ (deformation) is proportional to τ shear stress
Fluids (Newtonian)	dγ/dt (rate of deformation, strain rate) is proportional to τ shear stress

non-Newtonian fluids

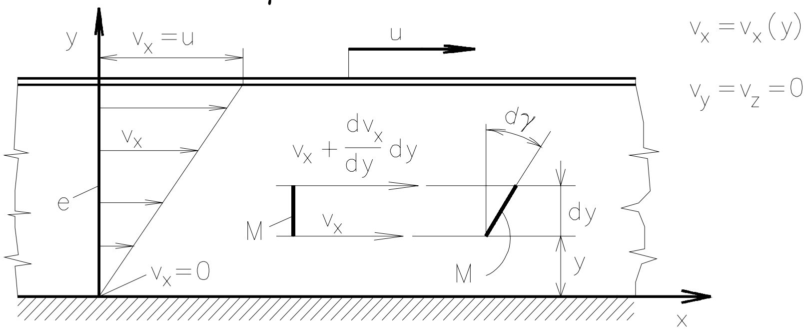
Fluids:

- no slip condition
- no change in internal structure at any deformation
- continuous deformation when shear stress exists
- no shear stress in fluids at rest

Viscosity

Velocity distribution: line or surface connecting the tips of velocity vectors the foot-end of which lies on a straight line or on a plane.

Turn of the bar M: dγ

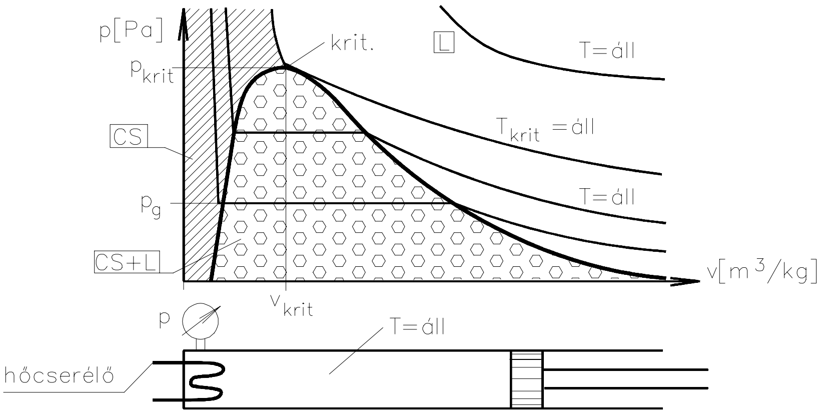


$$\frac{d\gamma}{dt} = \frac{dv_x}{dy}. \qquad \boxed{\tau_{xy} = \mu\frac{dv_x}{dy} = \mu\frac{d\gamma}{dt}.}$$ Newton's law of viscosity

$$[\mu] = [\tau]\left[\frac{dy}{dv_x}\right] = \frac{kgm}{s^2m^2}\frac{m}{m/s} = \frac{kg}{ms}.$$ Dynamic viscosity

$$\boxed{\nu = \frac{\mu}{\rho}} \; [m^2/s]$$ Kinematic viscosity

Compression of water vapor



heat exchanger T=const

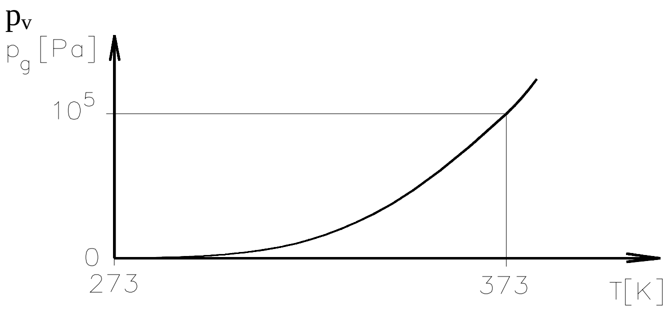
If T >> T_{krit} : gas O_2 and N_2 $\rightarrow T_{krit}$ 154 [K] and 126 [K]

$$pv = \boxed{\frac{p}{\rho} = RT}$$ ideal-gas law

where p[Pa], ρ [kg/m³], T [K], $R = R_u/M$, R_u = 8314.3 J/kmol/K universal gas constant, M kg/kmol molar mass, for air: M=29 [kg/kmol], therefore R=287J/kg/K.

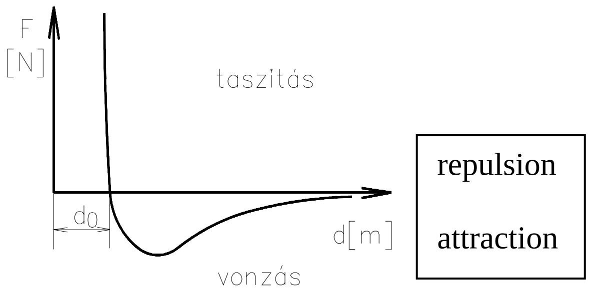
Cavitation

saturated steam pressure (vapor pressure) - temperature. Water 15 ^{0}C, p_v = 1700Pa, 100 ^{0}C, p_v = 1.013*10^5 Pa standard atmospheric pressure



Cavitation erosion

Interactions between molecules (attraction and repulse)



Comparison f liquids and gases

	liquids	gases
distance between molecules	small ≅ d_0	large ≅ $10d_0$
role of interactions of molecules	significant⇒ free surface	small ⇒ fill the available space
effect of change of pressure on the volume	small ⇒ 1000 bar causes 5% decrease in V	large⇒ in case of T=const V proportional to 1/p
cause of viscosity	attraction among molecules	momentum exchange among molecules
relation between viscosity and temperature pressure	T increases μ decreases independent	T increases μ increases independent

Comparison of real and perfect fluids

	real fluids	perfect fluids
viscosity	viscous	inviscid
density	compressible	incompressible
structure	molecular	continuous

3. Description of flow field

Scalar fields

Density $\rho_v = \lim_{\Delta V \Rightarrow \varepsilon^3} \frac{\Delta m}{\Delta V}\left[kg/m^3\right]$ ΔV incremental volume $\varepsilon >> \lambda$ (mean free path)

continuum $\rho=\rho(\underline{r},t)$ $\rho=\rho(x,y,z,t)$

Pressure

$p = |\Delta \underline{F}| / |\Delta \underline{A}|$ [N/m^2], [Pa].

$p=p(\underline{r},t)$, $p=p(x,y,z,t)$

Temperature

$T=T(\underline{r},t)$

Vector fields

Velocity

$\underline{v} = \underline{v}(\underline{r}, t)$ Eulerian description of motion

Fields (of force) $[\underline{g}] = N/kg = m/s^2$.

gravity field: $\underline{g} = -g_g \underline{k}$ $g_g = 9.81$ N/kg

field of inertia: accelerating coordinate system ($\underline{a} = a\underline{i}$) $\underline{g_t} = -a\underline{i}$.

centrifugal field: rotating coordinate system $\underline{g_c} = \underline{r}\omega^2$

Characterization of fields

Characterization of scalar fields:

$$\text{grad}p = \frac{\partial p}{\partial x}\underline{i} + \frac{\partial p}{\partial y}\underline{j} + \frac{\partial p}{\partial z}\underline{k} = \frac{\partial p}{\partial \underline{r}}$$ gradient vector

4 characteristics of the vector:

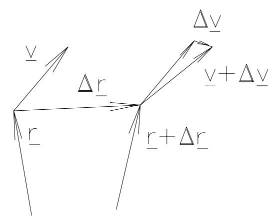
it is parallel with the most rapid change of p
it points towards increasing p
its length is proportional to the rate of the change of p
it is perpendicular to p = constant surfaces

Change of a variable: e.g. increment of pressure

$$\Delta p = p_B - p_A \cong \text{grad}p\Delta\underline{s} = \frac{\partial p}{\partial x}\Delta x + \frac{\partial p}{\partial y}\Delta y + \frac{\partial p}{\partial z}\Delta z$$

Characterization of vector fields: $\underline{v} = v_x \underline{i} + v_y \underline{j} + v_z \underline{k} = \underline{v}(\underline{r}, t)$.

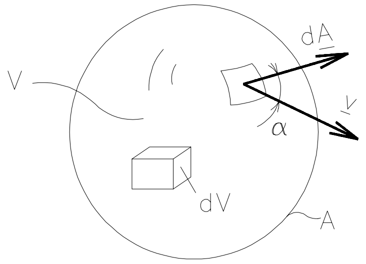
$v_x = v_x(x, y, z, t), v_y = v_y(x, y, z, t), v_z = v_z(x, y, z, t)$. vector field = 3 scalar fields



$$\Delta v_x \cong \operatorname{grad} v_x \Delta \underline{r} = \frac{\partial v_x}{\partial x}\Delta x + \frac{\partial v_x}{\partial y}\Delta y + \frac{\partial v_x}{\partial z}\Delta z .$$

$$\Delta \underline{v} \cong \begin{bmatrix} \frac{\partial v_x}{\partial x}\Delta x + \frac{\partial v_x}{\partial y}\Delta y + \frac{\partial v_x}{\partial z}\Delta z \\ \frac{\partial v_y}{\partial x}\Delta x + \frac{\partial v_y}{\partial y}\Delta y + \frac{\partial v_y}{\partial z}\Delta z \\ \frac{\partial v_z}{\partial x}\Delta x + \frac{\partial v_z}{\partial y}\Delta y + \frac{\partial v_z}{\partial z}\Delta z \end{bmatrix}$$

Divergence: $\operatorname{div}\underline{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$,



$$dq_v = \underline{v} d\underline{A} = |\underline{v}||d\underline{A}|\cos\alpha \left[m^3/s\right]$$

$$\boxed{\int_A \underline{v} d\underline{A} = \int_V \operatorname{div}\underline{v} dV}$$ Gauss-Osztrogradszkij theorem

Rotation, vorticity: $\operatorname{rot}\underline{v} = \underline{\nabla} \times \underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} = \begin{bmatrix} \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \\ \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \end{bmatrix}$

$$\boxed{\operatorname{rot}\underline{v} = 2\underline{\Omega}}.$$

$$\boxed{\Gamma = \oint_G \underline{v} d\underline{s} = \int_A \operatorname{rot}\underline{v} d\underline{A}}$$ Stokes theorem

Potential flow

$\underline{v} = \text{grad}\varphi$ condition: $\Gamma = \oint_G \underline{v} d\underline{s} = 0$, or $\text{rot}\underline{v} = 0$

Example: fields of force

for gravity force $\oint_G \underline{g} d\underline{s} = 0$ work of the field

U [m^2/s^2] potential of the field

$\boxed{\underline{v} = -\text{ grad U}}$

gravity field: $\underline{g} = -g_g \underline{k}$ $\boxed{U_g = g_g z + \text{konst.}}$

field of inertia: accelerating coordinate system ($\underline{a} = a\,\underline{i}$) $\underline{g_t} = -a\underline{i}$ $\boxed{U_t = ax + \text{konst.}}$

centrifugal field: rotating coordinate system $\underline{g_c} = \underline{r}\omega^2$ $\boxed{U_c = -\frac{r^2\omega^2}{2} + \text{kost.}}$

4. Kinematics

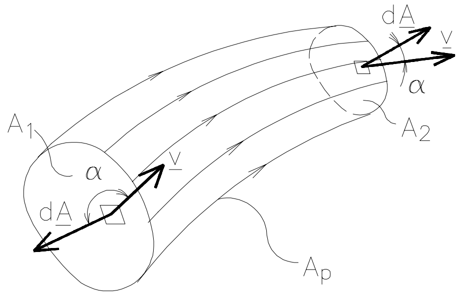
Definitions

Pathline: loci of points traversed by a particle (photo: time exposure)

Streakline: a line whose points are occupied by all particles passing through a specified point of the flow field (snapshot). Plume arising from a chimney, oil mist jet past vehicle model

Streamline: $\underline{v}$ x d$\underline{s}$ = 0 velocity vector of particles occupying a point of the streamline is tangent to the streamline.

Stream surface, stream tube: no flow across the surface.



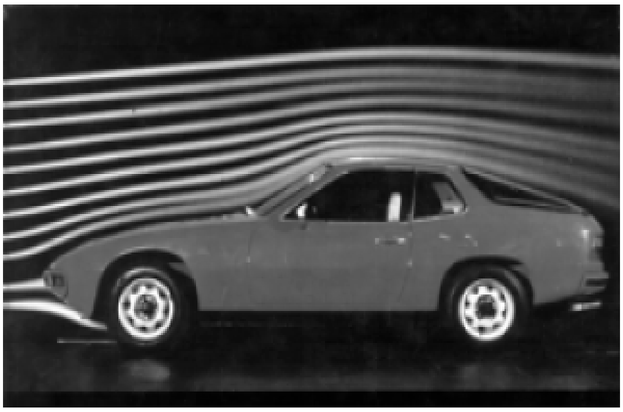
Time dependence of flow: Unsteady flow: **$\underline{v} = \underline{v}(\underline{r}, t)$** Steady flow: **$\underline{v} = \underline{v}(\underline{r})$**

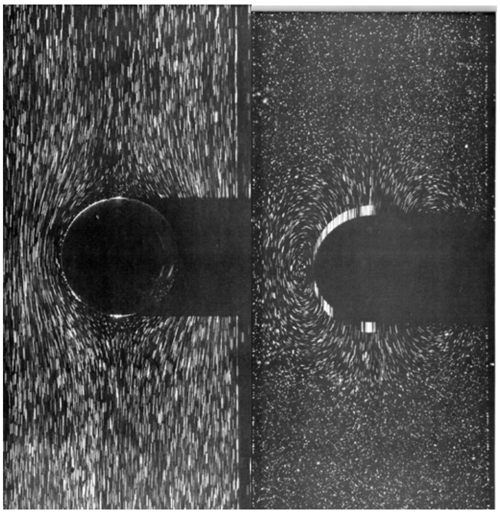
In some cases the time dependence can be eliminated through transformation of coordinate system.

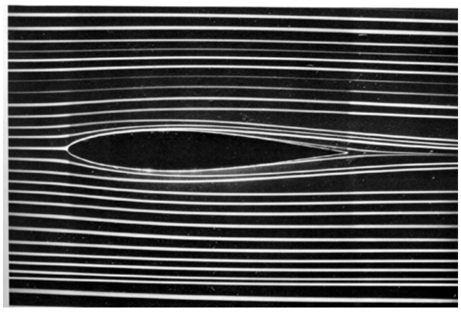
In steady flows pathlines, streaklines and streamlines coincide, at unsteady flows in general not.

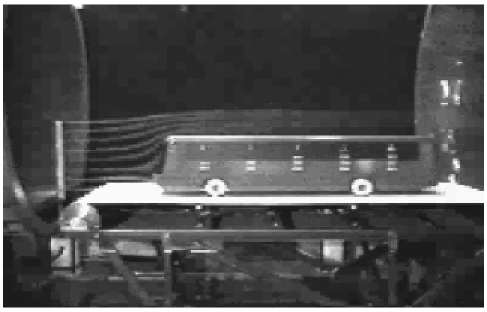
Flow visualization:
quantitative and/or qualitative information

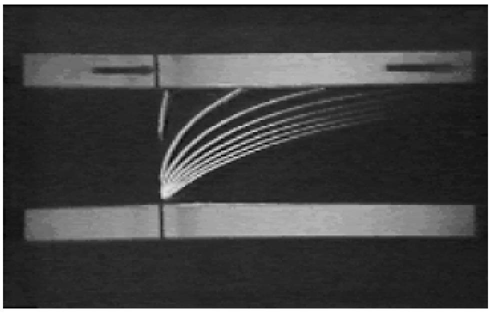
a) Transparent fluids, light-reflecting particles (tracers) moving with the fluid: particles of the same density, or small particles (high aerodynamic drag). Oil mist, smoke, hydrogen bubbles in air and in water, paints, plastic spheres in water, etc. PIV (Particle Image Velocymetry), LDA Laser Doppler Anemometry),
b) Wool tuft in air flow shows the direction of the flow.

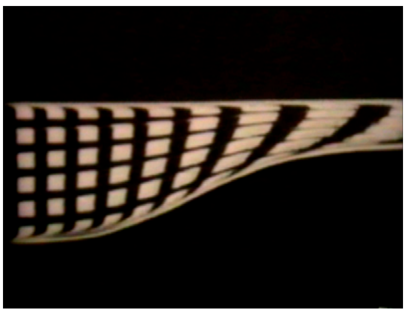


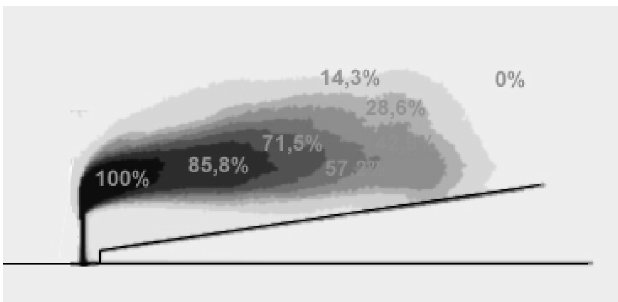


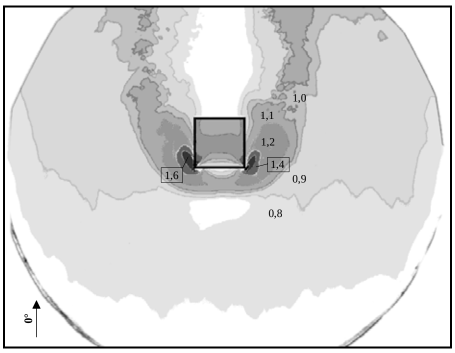




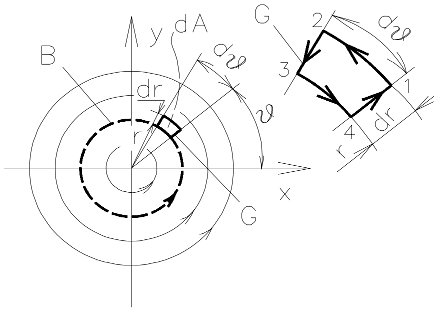








Irrotational (potential) vortex



Concept pf two-dimensional (2D), plane flows:

$$v_z = 0 \quad \text{and} \quad \frac{\partial v_x}{\partial z} = \frac{\partial v_y}{\partial z} = 0.$$

Because of continuity consideration at vortex flow v = v(r) **v(r) = ?**

Calculation of rot$\underline{v}$ using Stokes theorem: $\boxed{\Gamma = \oint_G \underline{v}d\underline{s} = \int_A \text{rot}\underline{v}d\underline{A}}$

$$\oint_G \underline{v}d\underline{s} = \int_1^2 \underline{v}d\underline{s} + \underbrace{\int_2^3 \underline{v}d\underline{s}}_{=0} + \int_3^4 \underline{v}d\underline{s} + \underbrace{\int_4^1 \underline{v}d\underline{s}}_{=0}$$

Since v⊥ds at 2nd and 4th integrals, and at 1st and 3rd integral $\underline{v}$ and d$\underline{s}$ include an angle of 0^0 and 180^0:

$$\oint_G \underline{v}d\underline{s} = (r+dr)d\vartheta v(r+dr) - rd\vartheta v(r)$$

Since

$$v(r+dr) = v(r) + \frac{dv}{dr}dr$$

after substitution

$$\oint_G \underline{v}d\underline{s} = rd\vartheta\frac{dv}{dr}dr + drd\vartheta v(r) + \underbrace{drd\vartheta\frac{dv}{dr}dr}_{\approx 0}$$

In plane flow only (rot$\underline{v}$)$_z$ differs from 0.

$$\int_{dA} rot\underline{v}d\underline{A} = (rot\underline{v})_z rd\vartheta dr$$

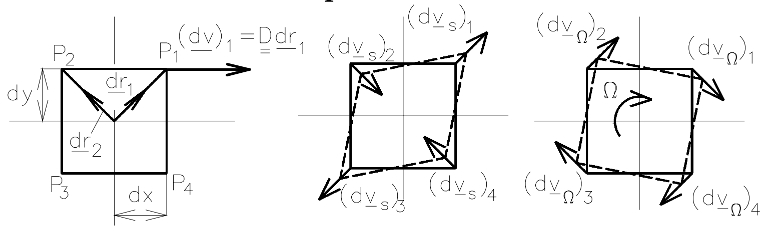
$$\boxed{(rot\underline{v})_z = \frac{dv}{dr} + \frac{v}{r}}.$$

Example: $v = \omega \cdot r \Rightarrow (rot\underline{v})_z = 2\omega$

In case of rot$\underline{v}$ = 0

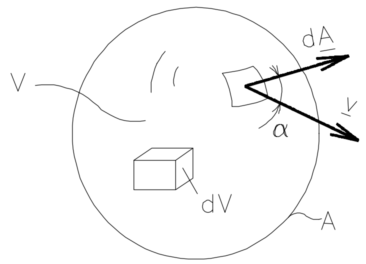
$\frac{dv}{v} = -\frac{dr}{r} \Rightarrow \ln v = -\ln r + \ln \text{Konst.} \Rightarrow \boxed{v = \frac{K}{r}}$. Velocity distribution in an irrotational (potential) vortex.

Motion of a small fluid particle



The motion of a FLID particle can be put together from **parallel shift, deformation and rotation.** In case of potential flow no rotation occurs.

5. Continuity equation



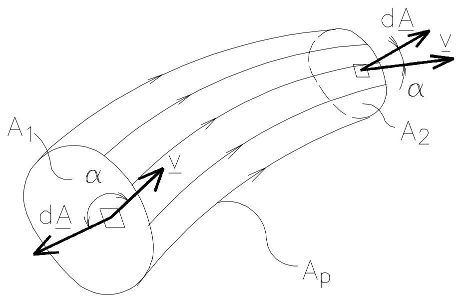
$dq_m = \rho \underline{v} d\underline{A} = \rho |\underline{v}| |d\underline{A}| \cos\alpha \ [kg/s]$

integral form of continuity equation: $\boxed{\int_A \rho \underline{v} d\underline{A} + \int_V \frac{\partial \rho}{\partial t} dV = 0}$

differential form: $\boxed{\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \underline{v}) = 0}$, if the flow is steady: $\underline{v} = \underline{v}(\underline{r}) \Rightarrow$ $\boxed{\mathrm{div}(\rho \underline{v}) = 0}$,

if the fluid is incompressible ρ= const. $\boxed{\mathrm{div}\underline{v} = 0}$

Application of continuity equation for a stream tube
Steady flow, no flow across the surface.



Integral form of continuity equation for steady flow: $\int_A \rho \underline{v} d\underline{A} = 0$. "A" consists of the mantle A_p ($\underline{v} \perp d\underline{A}$) and A_1 and A_2 in- and outflow cross sections. $\int_{A_1} \rho \underline{v} d\underline{A} + \int_{A_2} \rho \underline{v} d\underline{A} = 0$. Since $\underline{v} d\underline{A} = |\underline{v}| |d\underline{A}| \cos\alpha$, $\int_{A_1} \rho |\underline{v}| |d\underline{A}| \cos\alpha + \int_{A_2} \rho |\underline{v}| |d\underline{A}| \cos\alpha = 0$ Assumptions: over A_1 and A_2 ($v \perp A$) and over A_1 $\rho = \rho_1$ =const., over A_2 $\rho = \rho_2$ =const'. $\rho \bar{v} A$ = Const., where $\bar{v}$ mean velocity at changing cross section of a pipeline: $\boxed{\rho_1 \bar{v}_1 A_1 = \rho_2 \bar{v}_2 A_2}$ $\Rightarrow$ $\bar{v}_2 = \bar{v}_1 \dfrac{\rho_1 D_1^{\,2}}{\rho_2 D_2^{\,2}}$

6. Hydrostatics

Static fluid: forces acting on the mass (e.g. gravity) and forces acting over the surface (forces caused by pressure and ~~shear stresses~~) balance each other (no acceleration of fluid).

$$\rho \, dx \, dy \, dz \, g_x + dy \, dz \, p(x) - dy \, dz \left(p(x) + \frac{\partial p}{\partial x} dx \right) = 0$$

$\rho \, g_x = \dfrac{\partial p}{\partial x}$ $\Rightarrow$ $\boxed{\text{grad } p = \rho \, \underline{g}}$ fundamental equation of hydrostatics.

<u>Assumption</u>: $\underline{g} = -\text{grad } U$ (potential field of force)

$\text{grad } p = -\rho \text{ grad } U$ $\Rightarrow$ p=const. surfaces coincide with U = Const. (equipotential surfaces)

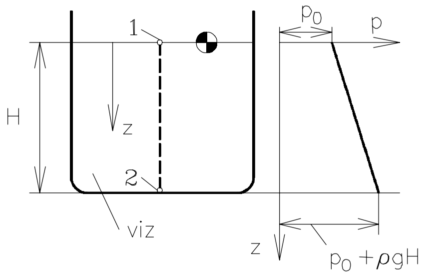
The surface of a liquid coincides with one of the U = Const. equipotential surfaces $\Rightarrow$ the surface is perpendicular to the field of force.

<u>Assumptions</u> $\underline{g} = -\text{grad } U$ (potential field of force), ρ = const. (incompressible fluid)

$$\frac{1}{\rho} \text{grad } p = \text{grad } \frac{p}{\rho} = -\text{grad } U \Rightarrow \text{grad} \left(\frac{p}{\rho} + U \right) = 0 \Rightarrow \boxed{\frac{p}{\rho} + U = \text{const.}}$$

$\boxed{\dfrac{p_1}{\rho_1} + U_1 = \dfrac{p_2}{\rho_2} + U_2}$ incomplete Bernoulli equation

Pressure distribution in a static and accelerating tank



$\underline{g}_g = g\underline{k}$, where $g = 9.81 \, \text{N/kg}$. $\dfrac{\partial p}{\partial x}\underline{i} + \dfrac{\partial p}{\partial y}\underline{j} + \dfrac{\partial p}{\partial z}\underline{k} = \rho g \underline{k}$ $dp/dz = \rho g$, ρ=áll. $p = \rho g z$+Const.

If $z = 0$, then $p = p_0$. $\Rightarrow$ $\text{Const.} = p_0$ $\Rightarrow$ $\boxed{p = p_0 + \rho g z}$. In z = H point $p = p_0 + \rho g H$

$\boxed{\dfrac{p_1}{\rho_1} + U_1 = \dfrac{p_2}{\rho_2} + U_2}$ point 1 on the surface =0), point 2 at the bottom (z = H). At z coordinate pointing downwards $U = -gz$, $p_1 = p_0, z_1 = 0,$ $p_2 = ?, z_2 = H$.

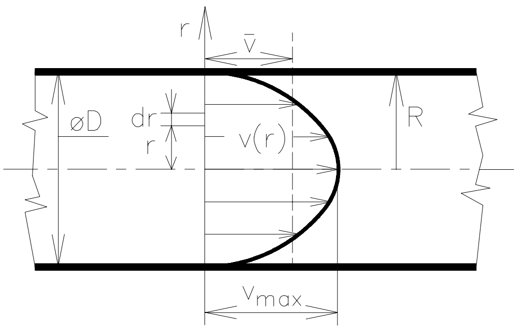
$\boxed{p_2 - p_0 = \rho g H}$

If the tank accelerates upwards, the fluid is static only in an upwards accelerating coordinate system. Here additional (inertial) field of force should be considered: $g_i = a\,\underline{k}$

$U_i = -az \quad U = U_g + U_i = -(g+a)z$. After substitution:

$\boxed{p_2 - p_0 = \rho(g+a)H}$

7. Calculation of mean velocity in a pipe of circular cross section



$\bar{v}$ = ? mean velocity

In cross section of diameter D the velocity distribution is described by a paraboloid. The difference of v_{max} and v(r) depends on the nth power of r $v(r) = v_{max}\left[1-(r/R)^n\right]$.

Mean velocity: $\bar{v} = \frac{4q_v}{D^2\pi}[m/s]$ where $q_v\left[m^3/s\right]$ is the flow rate.

The flow rate through an annulus of radius r thickness dr, cross section 2rπdr is dq_v = 2rπ v($\underline{r}$)dr $\Rightarrow$ $q_v = \int_0^R 2r\pi v_{max}\left[1-(r/R)^n\right] dr$.

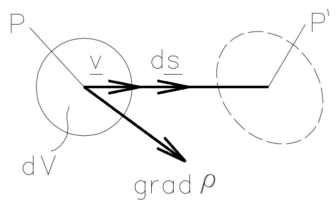
Integration yields: $q_v = R^2 \pi v_{max} \frac{n}{n+2}$, so the mean velocity is:

$\bar{v} = \frac{n}{n+2} v_{max}$.

In case of paraboloid of 2nd degree (n = 2) the mean velocity is half of the maximum velocity.

8. Local and convective change of variables

$$\frac{\partial\rho}{\partial t} + \mathrm{div}(\rho\underline{v}) = 0 \Rightarrow \frac{\partial\rho}{\partial t} + \underline{v}\,\mathrm{grad}\,\rho + \rho\,\mathrm{div}(\rho\underline{v}) = 0$$



In point P the velocity is $\underline{v}$, the variation of density in space is characterized by gradρ. Unsteady flow: $\partial\rho/\partial t \neq 0$. Variation of density dρ in time dt?

Two reasons for variation of ρ:

a) Because of time dependence of density ($\partial\rho/\partial t \neq 0$), the variation of density in point P: $d\rho_l = \frac{\partial\rho}{\partial t}dt$

b) In dt time the fluid particle covers a distance $d\underline{s} = \underline{v}dt$ and gets in P' point, where the density differs $d\rho_c = \text{grad}\rho\ \underline{ds} = \text{grad}\rho\ \underline{v}\ dt$ from that of in point P.

$d\rho_l$ local variation of density (only in unsteady flows)

$d\rho_c$ convective variation of density is caused by the flow and the spatial variation of the density

The substantial variation of the density is time dt: $d\rho = d\rho_l + d\rho_c = \frac{\partial\rho}{\partial t}dt + \underline{v}\ \text{grad}\rho\ dt$,

The variation in time unit: $\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \underline{v}\text{grad}\rho \Rightarrow \frac{d\rho}{dt} + \rho\text{div}\underline{v} = 0$

9. Acceleration of fluid particles

The variation of v_x in unit time.

$$\frac{dv_x}{dt} = \frac{\partial v_x}{\partial t} + \underline{v}\,\text{grad}\,v_x \cdot$$

Acceleration of fluid particle in x direction.

The first term: local acceleration, the second term: convective acceleration.

$$\frac{dv_x}{dt} = \frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}$$
$$\frac{dv_y}{dt} = \frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}$$
$$\frac{dv_z}{dt} = \frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}$$

Determining the differential of $\underline{v}(\underline{r},t)$: $d\underline{v} = \frac{\partial\underline{v}}{\partial t}dt + \frac{\partial\underline{v}}{\partial\underline{r}}\frac{\partial\underline{r}}{\partial t}dt$. Referring $d\underline{v}$ to unit time, i.e. dividing it by dt: $\frac{d\underline{v}}{dt} = \frac{\partial\underline{v}}{\partial t} + \frac{\partial\underline{v}}{\partial\underline{r}}\frac{\partial\underline{r}}{\partial t}$, where $\frac{\partial\underline{r}}{\partial t} = \underline{v}$

Local acceleration is different from 0 if the flow is unsteady. The convective acceleration exists, if the magnitude and/or direction of flow alter in the direction of the motion of the fluid.

The formula for acceleration can be transformed:

$$\frac{d\underline{v}}{dt} = \frac{\partial\underline{v}}{\partial t} + \text{grad}\frac{v^2}{2} - \underline{v}\times\text{rot}\underline{v}.$$