

Turbulence
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Wall

LES

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Filtering

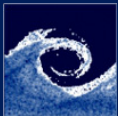
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Turbulence modelling III.

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Wall boundary conditions

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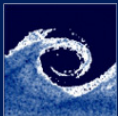
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- Both k and ε or ω require boundary conditions at the walls
- Before introducing the boundary conditions and the approximate boundary treatment techniques, some theory about wall boundary layers is required.

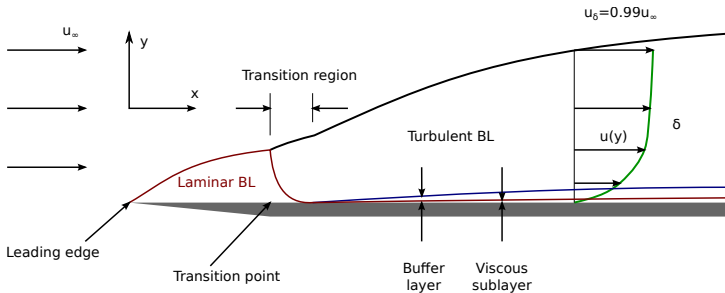


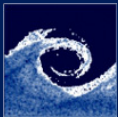
Boundary Layer

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Channel flow

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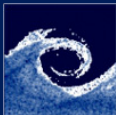
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Characteristics

- Flow between two infinite plates \Rightarrow fully developed
- Channel half width: δ
- Bulk velocity: $U_b \stackrel{\text{def}}{=} \frac{1}{\delta} \int_0^{\delta} \bar{u} dy$
- Bulk Reynolds number: $Re_b \stackrel{\text{def}}{=} \frac{U_b 2\delta}{\nu}$
- $Re_b > 1800$ means turbulence



Channel flow (contd.)

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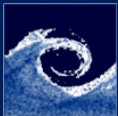
Streamwise averaged momentum equation:

$$0 = \underbrace{\nu d_y^2 \bar{u}}_{d_y \tau_l} - \underbrace{d_y \overline{u'v'}}_{d_y \tau_t} - \frac{1}{\rho} \partial_x \bar{p} \quad (1)$$

The pressure gradient ($d_x \bar{p}_w$) is balanced by the two shear stresses: $\tau = \tau_l + \tau_t$

Its distribution is linear:

$$\tau(y) = \tau_w \left(1 - \frac{y}{\delta}\right) \quad (2)$$



Channel flow (contd.)

Two type of shear stresses

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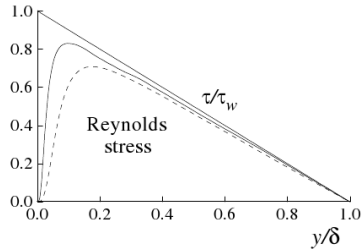
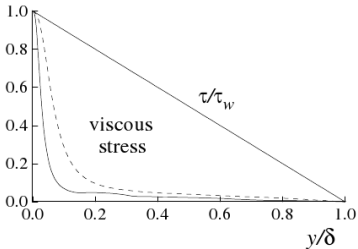
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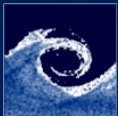
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The two shear stresses

- The viscous stress is dominant at the wall
- Turbulent stress is dominant far from the wall
- Both stresses are important in an intermediate region



Two scales of the flow at the wall

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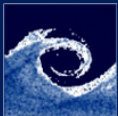
Definitions

- Friction velocity: $u_\tau \stackrel{\text{def}}{=} \sqrt{\frac{\tau_w}{\rho}} = \sqrt{-\frac{\delta}{\rho} d_x \overline{p_w}}$
- Friction Reynolds number: $Re_\tau \stackrel{\text{def}}{=} \frac{u_\tau \delta}{\nu} = \frac{\delta}{\delta_\nu}$
- Viscous length scale: $\delta_\nu \stackrel{\text{def}}{=} \frac{\nu}{u_\tau}$

General law of the wall can be characterised:

$$d_y \bar{u} = \frac{u_\tau}{y} \Phi\left(\frac{y}{\delta_\nu}, \frac{y}{\delta}\right) \quad (3)$$

Φ is a function to be determined!



Law of the wall

In wall proximity

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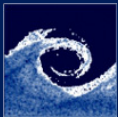
It can be assumed that only the wall scale plays a role in the near wall region:

$$d_y \bar{u} = \frac{u_\tau}{y} \Phi_I \left(\frac{y}{\delta_\nu} \right) \quad \text{for } y \ll \delta \quad (4)$$

Wall non-dimensionalisation \square^+

$$u^+ \stackrel{\text{def}}{=} \frac{\bar{u}}{u_\tau} \quad (5)$$

$$y^+ \stackrel{\text{def}}{=} \frac{y}{\delta_\nu} = \frac{y u_\tau}{\nu} \quad (6)$$



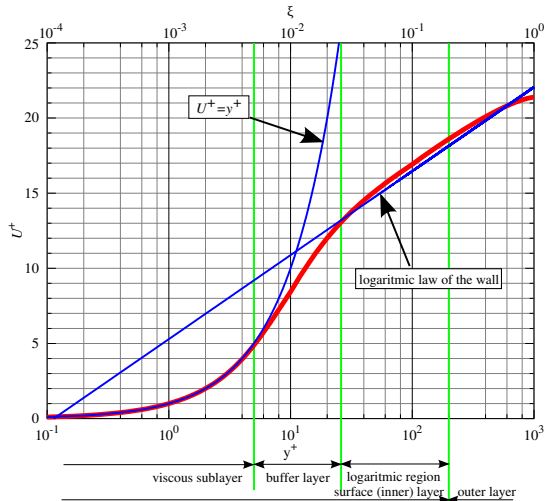
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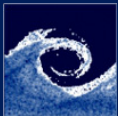
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Law of the wall

Velocity

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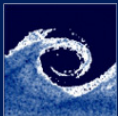
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Viscous sub-layer

- Only τ_l counts
- $u^+ = y^+$
- for $y^+ < 5$

Logarithmic layer

- Viscosity is not in the scaling
- $\Phi_I = \frac{1}{\kappa}$ for $y \ll \delta$ and $y^+ \gg 1$
- Log-law: $u^+ = \frac{1}{\kappa} \ln(y^+) + B$
 - From measurements: $\kappa \approx 0.41$ and $B \approx 5.2$



Law of the wall

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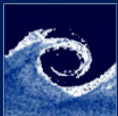
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Outer layer

- Φ depends only on y/δ
- In CFD we want to compute it for the specific cases! \Rightarrow
We do not deal with it.



Reynolds stress tensor at the wall

u_τ scaling

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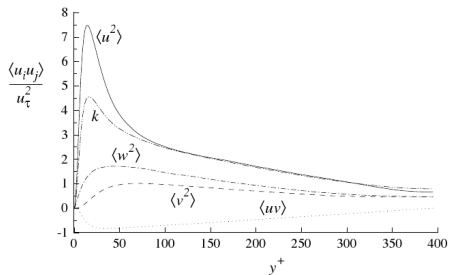
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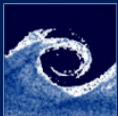
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Sharp peaks around $y^+ = 20$



Reynolds stress tensor at the wall

k scaling

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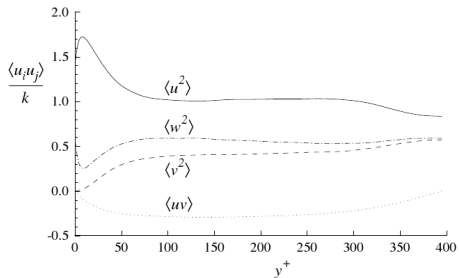
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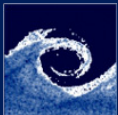
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A plateau is visible in the log law region.



TKE budget at the wall

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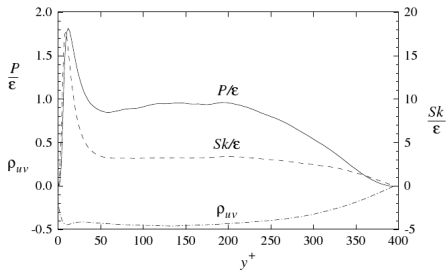
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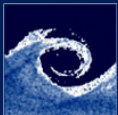
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- $P/\epsilon \approx 1$ in the log-law region
- $P/\epsilon \approx 1.8$ close to the wall



TKE budget at the wall

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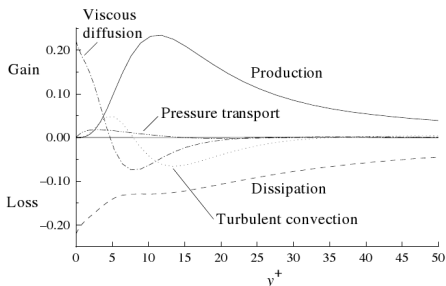
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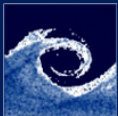
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- Turbulence is mainly produced in the buffer region ($5 < y^+ < 30$)
- Turbulence is viscous diffused to the wall
- Turbulence is strongly dissipated at the wall
- Conclusion: $\varepsilon = \nu d_{y^+}^2 u \quad @ \ y = 0$



Numerical treatment of the wall layer, actual BC's

Low Re treatment

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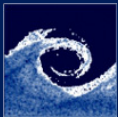
In this treatment the complete boundary layer is resolved numerically

When to do?

- Low Reynolds number flow, where resolution is feasible
- If boundary layer is not simple, can not be described by law of the wall

How to do?

- Use a turbulence model incorporating near wall viscous effects
- Use appropriate wall resolution ($y^+ < 1$)



Numerical treatment of the wall layer, actual BC's

High Re treatment

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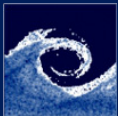
In this treatment the first cell incorporates the law of the wall

When to do?

- High Reynolds number flow, where it is impossible to resolve the near wall region
- If boundary layer is simple, can be well described by law of the wall

How to do?

- Use a turbulence models containing law of the wall BC
- Use appropriate wall resolution ($30 < y^+ < 300$)



Wall functions for RANS in practice (U , ν_t)

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- Velocity at the wall:
 - Dirichlet BC, no slip: $U(y=0) = 0$
- Turbulent viscosity (for the wall adjacent cells):
 - Wall shear-stress (friction velocity) by definition:

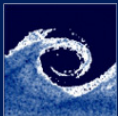
$$\tau_w = u_\tau^2 = \nu_t \frac{\partial U}{\partial y}$$

- Laminar case ($y^+ \leq y_{lam}^+$):

$$U^+ = y^+ \rightarrow \nu_t = 0$$

- Turbulent case ($y^+ > y_{lam}^+$):

$$U^+ = \kappa \ln(Ey^+) \rightarrow \nu_t = \nu \left(\frac{\kappa y^+}{\ln(Ey^+)} - 1 \right)$$



Wall functions for RANS in practice (k , ϵ , P_k)

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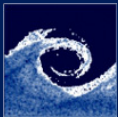
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- Turbulent kinetic energy (for the wall):
 - Neumann BC: $\frac{\partial k}{\partial y} = 0$
- Turbulent kinetic energy dissipation (for the wall adjacent cells):
 - Equilibrium assumption:

$$P_k = \nu_t \left(\frac{\partial U}{\partial y} \right)^2 = C_\mu \frac{k^2}{\epsilon} \left(\frac{\partial U}{\partial y} \right)^2 = \epsilon$$

- Implementation:

$$\epsilon = \frac{C_\mu^{0.75} k^{1.5}}{\kappa y} \quad \text{and} \quad P_k = (\nu + \nu_t) \left| \frac{\partial U}{\partial y} \right| \frac{C_\mu^{0.25} k^{0.5}}{\kappa y}$$



Numerical treatment of the wall layer, actual BC's

Clever laws

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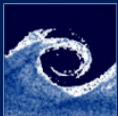
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The mixture of the two methods is developed:

- to enable the engineer not to deal with the wall resolution
- usually the mixture of the two method is needed, depending on actual position in the domain

Resolution requirements

At **any kind** of treatment the boundary layer thickness (δ) has to resolved by ≈ 20 cells to ensure accuracy.



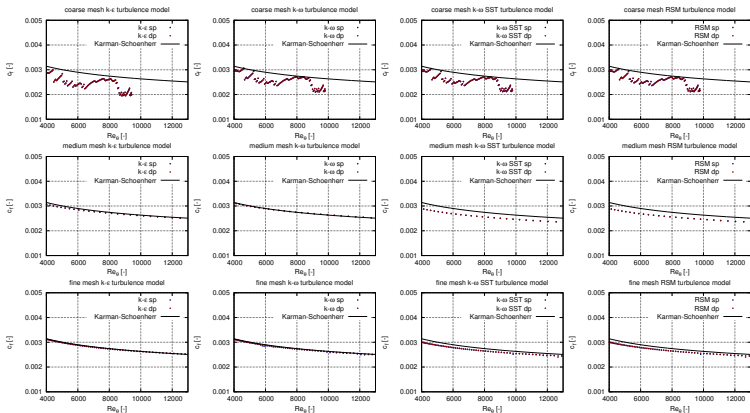
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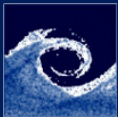
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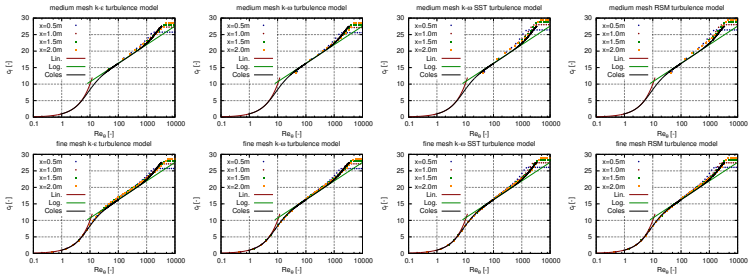
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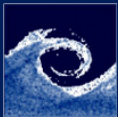
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Large-Eddy Simulation

Difference between modelling and simulation

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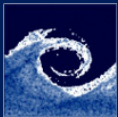
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Simulation

In the simulation the turbulence phenomena is actually resolved by a numerical technique, by solving the describing equations

Modelling

In the modelling of turbulence the effects of turbulence are modelled relying on theoretical and experimental knowledge. In the computation a reduced description of turbulence is carried out



Direct Numerical Simulation = DNS

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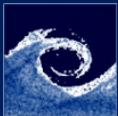
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The NS equations (describing completely the turbulence phenomena) are solved numerically

Difficulties

- The scales where the dissipation is effective are very small
 - The size of the smallest scales are Reynolds number dependent
- Simulation is only possible for academic situations (e.g.: HIT on $64 \cdot 10^9$ cells)



Concept of LES

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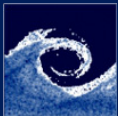
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Compromise between RANS and DNS

- RANS: feasible but inaccurate
- DNS: accurate but infeasible

The large scales are import to simulate

- The large scales of the turbulent flow are boundary condition dependent, they needs to be simulated
- The small scales of turbulence are more or less universal and can be modelled 'easily'
- The removal of the small scales form the simulation reduce the computational cost remarkably



Filtering

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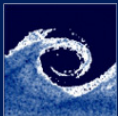
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How to develop the equations?

How to separate between large and small scales?

Spatial filtering, smoothing using a kernel function

$$\langle \varphi \rangle (x_j, t) \stackrel{\text{def}}{=} \int_V G_{\Delta}(r_i; x_j) \varphi(x_j - r_i, t) dr_i \quad (7)$$



Filtering kernel

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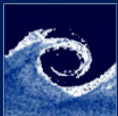
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- G_Δ is the filtering kernel with a typical size of Δ .
- G_Δ has a compact support (its definition set where the value is non-zero is closed) in its first variable
- To be the filtered value of a constant itself it has to be true:

$$\int_V G_\Delta(r_i; x_j) dr_i = 1 \quad (8)$$

- If $G_\Delta(r_i; x_j)$ is homogeneous in its second variable and isotropic in its first variable than $G_\Delta(|r_i|)$ is a function of only one variable



Filtering kernel

Examples

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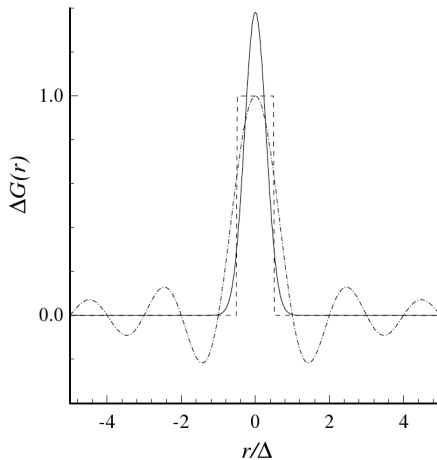
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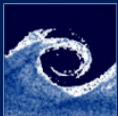
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Filtering

Physical space

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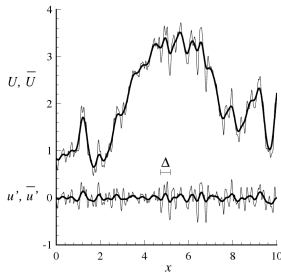
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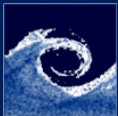
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Fluctuation:

$$\tilde{\varphi} \stackrel{\text{def}}{=} \varphi - \langle \varphi \rangle \quad (9)$$

$\langle \tilde{\varphi} \rangle \neq 0$, a difference compared to Reynolds averaging



Filtering

Spectral space

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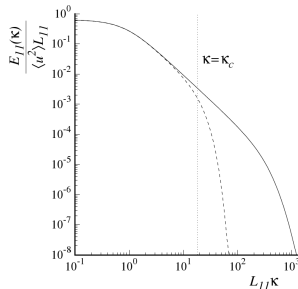
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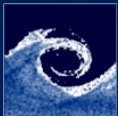
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Recall: the cutting wavenumber (κ_c), below which modelling is needed



Filtered equations

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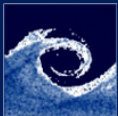
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- If using the previously defined (homogeneous, isotropic) filter
- Averaging and the derivatives commute (exchangeable)

$$\partial_i \langle u_i \rangle = 0 \quad (10)$$

$$\partial_t \langle u_i \rangle + \langle u_j \rangle \partial_j \langle u_i \rangle = -\frac{1}{\rho} \langle p \rangle + \nu \partial_j \partial_j \langle u_i \rangle - \partial_j \tau_{ij} \quad (11)$$

- 3D (because turbulence is 3D)
- unsteady (because the large eddies are unsteady)



Sub Grid Scale stress

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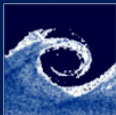
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τ_{ij} is called **Sub-Grid Scale stress SGS** from the times when filtering was directly associated to the grid

$$\tau_{ij} \stackrel{\text{def}}{=} \langle u_i u_j \rangle - \langle u_i \rangle \langle u_j \rangle \quad (12)$$

- It represents the effect of the filtered scales
- It is in a form a stress tensor
- Should be dissipative to represent the dissipation on the filtered small scale



Eddy viscosity model

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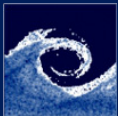
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- Same as in RANS

-

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2\nu_t \langle s_{ij} \rangle \quad (13)$$

- Relatively a better approach since the small scales are more universal
- Dissipative if $\nu_t > 0$.



Smagorinsky model

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III.

Miklós
Balogh

BC-s

Wall

LES

Concept

Filtering

BC-s

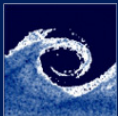
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$$\nu_t = (C_s \Delta)^2 |\langle S \rangle| \quad (14)$$

-

$$|\langle S \rangle| \stackrel{\text{def}}{=} \sqrt{2s_{ij}s_{ij}} \quad (15)$$

- C_s Smagorinsky constant to be determined
 - using spectral theory of turbulence
 - using validations on real flow computations
- Δ to be prescribed
 - Determine the computational cost (if too small)
 - Determine the accuracy (if too big)
 - 80% of the energy is resolved is a compromise



Scale Similarity model

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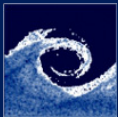
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Let us assume that the cuted small scales are similar to the kept large scales!

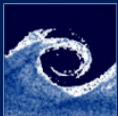
A logical model:

$$\tau_{ij} \stackrel{\text{def}}{=} \langle\langle u_i \rangle \langle u_j \rangle\rangle - \langle\langle u_i \rangle\rangle \langle\langle u_j \rangle\rangle \quad (16)$$



Properties

- It is not dissipative enough
- It gives feasible shear stresses (from experience)
- Logical to combine with Smag. model!



Dynamic approach

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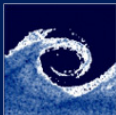
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- The idea is the same as in the scale similarity model
- The theory is more complicated
- Any model can be made dynamic
- Dynamic Smagorinsky is widely used (combining the two advantages)



Numerical issues

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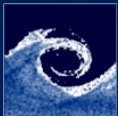
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Filtering

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- The spatial numerical schemes are used on the border of their capabilities (wave length = cell size), when $\Delta = h$ ($h =$ cell size)
- The numerical schemes remarkably influence the result
- Grid in-dependency as a function of h/Δ : practically impossible



Boundary Conditions

Periodicity

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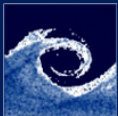
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Concept

Filtering

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- Periodicity is used to model infinite long domain
- The length of periodicity is given by the length scales of turbulence



Boundary Conditions

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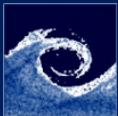
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Concept

Filtering

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- Much more difficult than in RANS
- Turbulent structures should be represented
 - Vortices should be synthesized
 - Separate precursor simulation to provide “real” turbulence



Boundary Conditions

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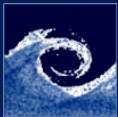
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$$y^+ \approx 1 \quad (17)$$

$$x^+ \approx 50 \quad (18)$$

$$z^+ \approx 10 - 20 \quad (19)$$



Results

Time averaged quantities

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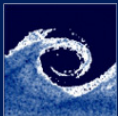
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Concept

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- Can be used similarly as results of RANS
- In a lucky situation it is more accurate than the RANS result, in case of bad use can be much more inaccurate



Results

Instantaneous structures

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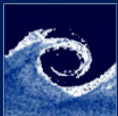
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Concept

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- The movement of the vortices can be tracked
- Enables the control of turbulence



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Thanks for your attention!