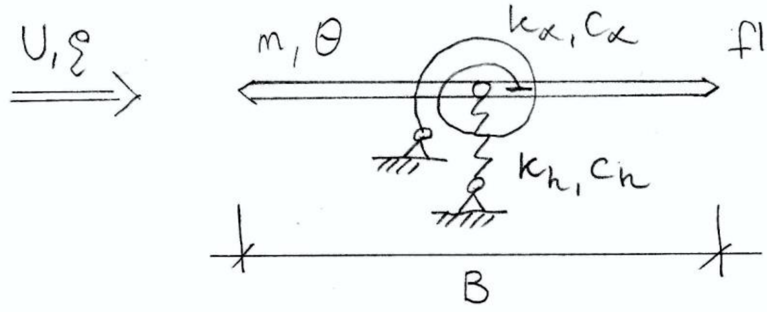
Flutter I.

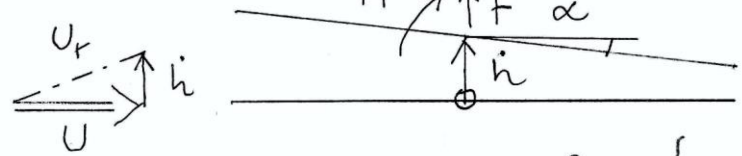
load models: – quasi-steady
– unsteady

quasy-steady approach:



flat-plate

$(C_L \cong 2\pi \beta_{eff})$



$$F = \pi \varrho U^2 B L \cdot \left\{\alpha - \frac{\dot h}{U}\right\}$$

$$M = \frac{1}{4}\pi \varrho U^2 B^2 L \cdot \underbrace{\left\{\alpha - \frac{\dot h}{U}\right\}}_{\beta_{eff}}$$

effective angle: $\beta_{eff} = \left\{\alpha - \frac{\dot h}{U}\right\}$

$$\begin{bmatrix} m & \\ & \Theta \end{bmatrix}\begin{bmatrix} \ddot h \\ \ddot\alpha \end{bmatrix} + \begin{bmatrix} c_h & \\ & c_\alpha \end{bmatrix}\begin{bmatrix} \dot h \\ \dot\alpha \end{bmatrix} + \begin{bmatrix} k_h & \\ & k_\alpha \end{bmatrix}\begin{bmatrix} h \\ \alpha \end{bmatrix} = \underline{Q}(\underline{x}, \underline{\dot x}) \qquad \underline{x} = \begin{bmatrix} h \\ \alpha \end{bmatrix}$$

$$= \pi \varrho U^2 B L \cdot \begin{Bmatrix} \alpha - \dot h / U \\ B/4\,(\alpha - \dot h/U) \end{Bmatrix}$$

$$\begin{bmatrix} m & \\ & \Theta \end{bmatrix}\begin{bmatrix} \ddot h \\ \ddot\alpha \end{bmatrix} + \begin{bmatrix} c_h + \pi \varrho U B L & 0 \\ 1/4\,\pi \varrho U B^2 L & c_\alpha \end{bmatrix}\begin{bmatrix} \dot h \\ \dot\alpha \end{bmatrix} + \begin{bmatrix} k_h & -\pi \varrho U^2 B L \\ 0 & k_\alpha - 1/4\,\pi \varrho U^2 B^2 L \end{bmatrix}\begin{bmatrix} h \\ \alpha \end{bmatrix} = \underline{0}$$

$$\underline{\underline{M}}\,\underline{\ddot x} + \underline{\underline{\hat C}}\,\underline{\dot x} + \underline{\underline{\hat K}}\,\underline{x} = \underline{0} \qquad \underline{x} = \underline{\tilde\Phi} \cdot e^{i\omega t}$$

$(\underline{\underline{C}}_S + \underline{\underline{C}}_{AE})$ $\qquad$ $(\underline{x} = \underline{\tilde\Phi} \cdot e^{\lambda t}$ alternatively)

$$\left[-\omega^2 \underline{\underline{M}} + i\omega \underline{\underline{\hat C}} + \underline{\underline{\hat K}}\right] \underline{\tilde\Phi} = \underline{0} \qquad \tilde\omega = a + ib$$

$$e^{i\tilde\omega t} = e^{iat} \cdot e^{i(ib)t} = e^{i\omega t} \cdot e^{-\xi t}$$

$\xi(U) = ?$

$\xi > 0$, stab.

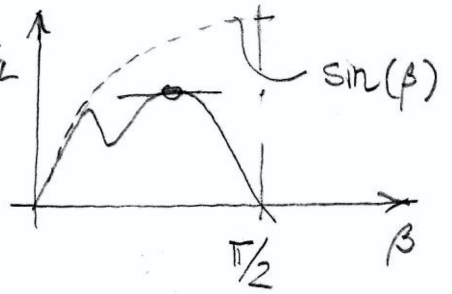
$\xi < 0$, instab.

$\xi = 0$, flutter onset

Hopf bifurcation

$$\underline{\underline{M}}\,\ddot{\underline{x}} + \underline{\underline{C}}\,\dot{\underline{x}} + \underline{\underline{K}}\,\underline{x} = \underline{Q}(\underline{x}, \dot{\underline{x}}, U)$$

$$\underline{Q} = \begin{Bmatrix} \pi \rho U^2 B L \cdot \sin(\alpha - \dot{h}/U) \\ 1/4\, \pi \rho U^2 B^2 L \cdot \sin(\alpha - \dot{h}/U) \end{Bmatrix}$$



$$\sin(\alpha - \dot{h}/U) \approx \left\{\alpha - \dot{h}/U\right\} + \left\{-\frac{1}{6}\alpha^3 + \frac{1}{2U}\alpha^2\dot{h}^2 - \frac{1}{2U^2}\alpha\dot{h}^2 + \frac{1}{6U^3}\dot{h}^3\right\}$$

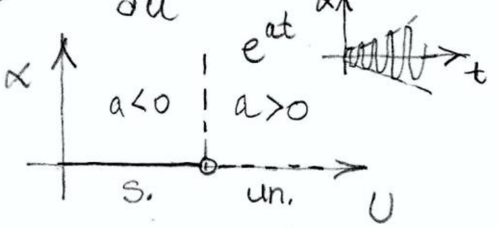
$$\underline{x} = \begin{bmatrix} h \\ \alpha \end{bmatrix};\quad \dot{\underline{x}} = \begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix};\quad \ddot{\underline{x}} = \begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix}$$

↓ Cauchy

$$\dot{\underline{z}} = \underline{\underline{A}}\,\underline{z} + \underline{g}(\underline{z}) \qquad z(3) = \alpha \qquad z(2) = \dot{h}$$

$$\sigma_0'(0) = \frac{\partial a(U)}{\partial u}$$

$$\left[\underline{\underline{A}}(U) - \lambda \underline{\underline{I}}\right]\tilde{\underline{\phi}} = \underline{0} \;\rightarrow\; \lambda = a + ib$$



↓ Hopf

$$\underline{\underline{T}} = \left[\mathrm{Re}(\tilde{\underline{\phi}}_1)\ \mathrm{Im}(\tilde{\underline{\phi}}_2)\ \mathrm{Re}(\tilde{\underline{\phi}}_2)\ \mathrm{Im}(\tilde{\underline{\phi}}_2)\right];\quad \underline{z} = \begin{bmatrix} h \\ \dot{h} \\ \alpha \\ \dot{\alpha} \end{bmatrix};\quad \tilde{\underline{\phi}} = \begin{bmatrix} \tilde{h} \\ \tilde{\dot{h}} \\ \tilde{\alpha} \\ \tilde{\dot{\alpha}} \end{bmatrix}$$

$\underline{z} = \underline{\underline{T}}\,\underline{u}$ (~ modal analisys)

$$\underline{\underline{T}}\,\dot{\underline{u}} = \underline{\underline{A}}\,\underline{\underline{T}}\,\underline{u} + \underline{g}(\underline{u}) \ / \underline{\underline{T}}^{-1} \rightarrow$$

$$\dot{\underline{u}} = \underbrace{\underline{\underline{T}}^{-1}\underline{\underline{A}}\,\underline{\underline{T}}}_{\underline{\underline{J}}}\,\underline{u} + \underbrace{\underline{\underline{T}}^{-1}\cdot\underline{g}(\underline{u})}_{\underline{b}}$$

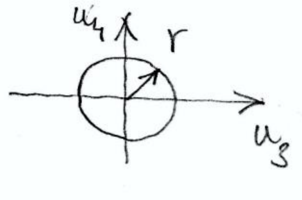
$$\underline{\underline{J}} = \left[\begin{array}{cc|cc} \sigma_1 & \omega_1 & \multicolumn{2}{c}{\underline{\underline{0}}} \\ -\omega_1 & \sigma_2 & & \\ \hline \multicolumn{2}{c|}{\underline{\underline{0}}} & 0 & \omega_2 \\ & & -\omega_2 & 0 \end{array}\right]$$

$$\underline{b} = \begin{bmatrix} - \\ - \\ \sum_{ij} a_{ij}\cdot u_3^i \cdot u_4^j \\ \sum_{ij} b_{ij}\cdot u_3^i \cdot u_4^j \end{bmatrix} \longrightarrow \delta(0) = \frac{1}{8}\cdot\left[\left\{3a_{03} + a_{12} + b_{21} + 3b_{03}\right\} + \frac{1}{\omega}\cdot\left\{\ \right\}\right]$$

$\delta(0) < 0$: supercritical
$\delta(0) > 0$: subcritical

↓ amplitudes

$$r = \sqrt{\frac{-\sigma_0'(0)}{\delta(0)}\cdot\left[U - U_{cr}\right]}$$



$$\alpha(U) = r(U)\cdot\left|\tilde{\phi}_2(3)\right|$$