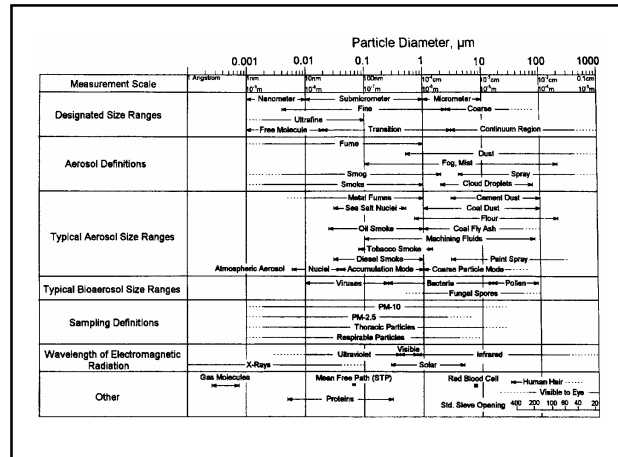


# Introduction to Aerodynamics of Aerosols and related Applications

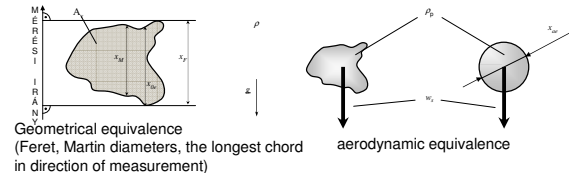
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## Aerosols

Definition of aerosols: nearly stable mixture of gas and particles (solid, liquid). Diameter of particles:  $0,01 \leq x [\mu\text{m}] \leq 50$   
Change of the characteristics of mixture caused  
- by diffusion and agglomeration of small particles as consequence of diffusion of small particles (Brownian motion)  
- by settling of larger particles.

In case of spherical particles the diameter is  $x [\mu\text{m}]$ . How to define the size of non-spherical particles? Geometrical, aerodynamic and optical equivalence.



Geometrical equivalence (Feret, Martin diameters, the longest chord in direction of measurement)

aerodynamic equivalence

## Types of particles in atmosphere in terms of their origin

The particles in the atmosphere can be divided in three groups:

### NATURAL PARTICLES:

- Aerosols of cosmic origin (in all the Earth  $10^7$  t/year)
- Inorganic aerosols (e.g. volcanic dust and ash, dust of deserts, sea salt)
- Organic aerosols (e.g. remains of vegetation, microbes, pollens)

TECHNOLOGICAL PARTICLE: They are products of technological processes like breakage, grinding, milling, classing, sizing, drying, condensation in gases.

### WASTE PARTICLES:

- Particles originating from settlements (roads, particles from buildings, soil-materials)
- Originating from production processes (coal mining, ore dressing, welding, exhaust gases of motors, polishing, grinding)
- Produced during combustion (carbon black, fly-ash)

### AIR POLLUTION IN HUNGARY

1.7% of the country is heavily, 6.2%-a moderately polluted by aerosols. 1/4-1/5th of population lives in these areas. Between 1980 and 2000 the aerosol emission was reduced by 75%, particularly the emission of industry and energy production, while that of traffic and transportation did not change significantly. The annual solid particle emission is in order of magnitude 100.000 tons. Industry 40%, inhabitants 25%-át, traffic and thermal power plants 13-13%.

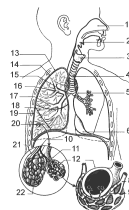
## Types of particles

- Dust:  $x \geq 0,2 [\mu\text{m}]$  perceptible by light microscope (diameter is larger than the wave length of light). Produced by breaking or attrition, abrasion, wearing of solid substances.
- Fume:  $x \leq 1 [\mu\text{m}]$  solid particles or fluid droplets originated from condensation or chemical reaction, in most cases chain-like structures. Produced at combustion, chemical processes.
- Mist: fluid droplets originated from steam condensation or by atomization, spraying. The mist droplets and the saturated steam are in equilibrium.

| AERA             | CONCENTRATION     |                    |                             |
|------------------|-------------------|--------------------|-----------------------------|
|                  | mg/m <sup>3</sup> | db/cm <sup>3</sup> | db/cm <sup>3</sup> > 0,1 µm |
| mountain         | 0.01              | 200 ÷ 1.000        | 2                           |
| flat, open area  | 0.02              | 2.000 ÷ 10.000     | 15                          |
| city             | 0,1 ÷ 0,4         | 5.000 ÷ 100.000    | 100                         |
| industrial area  | 0,2 ÷ 2           |                    | 1500                        |
| street, downtown | 1 ÷ 3             |                    | 200 ÷ 3000                  |

Dust dispersion (deposition of dust on ground, buildings, streets) in cities: 10-100 t/km<sup>2</sup>/month.

| TECHNOLOGIES                       | c(g/m <sup>3</sup> ) |
|------------------------------------|----------------------|
| Cement production                  | 5 ÷ 30               |
| Pulverised coal fired power plants | 15 ÷ 20              |
| Metallurgy of ferrous metals       | 10 ÷ 40              |
| Metallurgy of lead, lead chamber   | 5 ÷ 20               |
| Drying                             | 10 ÷ 100             |
| Bauxit grinding                    | 10 ÷ 30              |

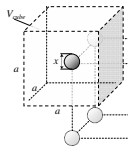


Upper respiratory tracts, bronchial tubes with mucous membrane and cells with cilia propelling mucus upwards (23 bifurcations), Air cells (Alveolus pulmos) for gas metabolism: O<sub>2</sub> CO<sub>2</sub>.

Dust fraction particularly harmful for lung:

$$0,5 \mu\text{m} \leq x \leq 5 \mu\text{m}$$

### Average distance between particles in gas



$$c = \frac{x^3 \cdot \pi \cdot \rho_p \cdot n}{6 \cdot a^3} \quad \frac{a}{x} = \sqrt[3]{\frac{\rho_p \cdot \pi}{6 \cdot c}}$$

$c$  [kg/m<sup>3</sup>] a concentration,  $a$  [m] average distance between particles,  $\rho$  [kg/m<sup>3</sup>] density of particles,  $n$  [db/m<sup>3</sup>] concentration by piece.

$$x = 3 \mu\text{m}$$

| c[g/m <sup>3</sup> ] | type of aerosol     | a/x | n [db/cm <sup>3</sup> ] |
|----------------------|---------------------|-----|-------------------------|
| 10                   | crude gas           | 47  | 350.000                 |
| 1                    | after pre-separator | 101 | 35.000                  |
| 0.1                  | after separator     | 218 | 3.500                   |

After transformation of the equation we get:

$$\frac{d\mathbf{v}}{dt} + \frac{c_p}{\rho_s} \frac{d\mathbf{u}_p}{dt} = \left(1 + \frac{c_p}{\rho_s}\right) \frac{\mathbf{g}}{\rho_s} - \frac{1}{\rho_s} \text{grad}p + \nu \Delta \mathbf{v}$$

The effect of particle phase on the flow field can be neglected, if

$\frac{c}{\rho} \ll 1$  and  $\frac{d\mathbf{u}_p}{dt}$  and  $\frac{d\mathbf{v}}{dt}$  are in the same order of magnitude

$$\text{so } \frac{c}{\rho} \frac{d\mathbf{u}_p}{dt} \ll \frac{d\mathbf{v}}{dt}$$

In this case the determination of flow field can be separated from that of the particle phase. Otherwise a PDE system consisting of 11 equations should be solved to determine distributions of 3+3 velocity components, gas pressure, particulate concentration and 3 components of aerodynamic force acting on particles. 3+3 momentum, 1+1 continuity and 3 eqs. for determining the components of  $\underline{F}$  aerodynamic force acting on particles.

### Particles in gas flow

Effect of particles on the gas flow

Navier-Stokes equation extended by term  $\underline{t}$  expressing the contribution of the particles to the forces acting on gas (the virtual mass of the particle is neglected):

$$\frac{\partial \mathbf{v}}{\partial t} + \text{grad} \frac{v^2}{2} - \nu \nabla^2 \mathbf{v} = \frac{\mathbf{g}}{\rho} - \frac{1}{\rho} \text{grad}p + \nu \Delta \mathbf{v} + \underline{t}$$

$\underline{t}$  [N/kg] is force to 1 kg gas from particles carried by the gas:

$$\underline{t} = -\frac{n\mathbf{F}}{\rho}$$

$n$  [particle/m<sup>3</sup>]: particle concentration by piece

$\mathbf{F}$  [N/particle] : aerodynamic force acting on a particle

$\rho$  [kg/m<sup>3</sup>] : gas density

### Drag force acting on spherical particle

Stokes: solution by linearization of NS equation and by neglecting the gravitational field of force:

$$0 = \frac{1}{\rho_s} \text{grad}p + \nu \Delta \mathbf{v} \Rightarrow \text{grad}p = -\mu \text{rotrot} \mathbf{v} \Rightarrow \text{div grad}p = \Delta p = -\mu \text{divrotrot} \mathbf{v} = 0$$

$\Delta p = 0$ ,  $\text{div} \mathbf{v} = 0$  equations were solved at boundary conditions:  $r = R$ ,  $\mathbf{v} = 0$ , and  $r \rightarrow \infty$ ,  $\mathbf{v} \rightarrow \mathbf{v}_\infty$ .

$$\mathbf{F} = 3\pi\mu x \mathbf{w} \quad c_c = \frac{|\mathbf{F}|}{\frac{\rho}{2} v^2 \frac{x^2 \pi}{4}} \quad Re_p < 1 \quad c_c = \frac{24}{Re_p} \quad Re_p = \frac{v x \rho}{\mu}$$

$$\text{Oseen's less drastic linearization} \quad v_\infty \frac{\partial v_x}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 v_x}{\partial x^2} + \dots \right)$$

$$Re_p < 5 \quad c_c = \frac{24}{Re_p} \left( 1 + \frac{3}{16} Re_p \right)$$

$$3 < Re_p < 400 \quad c_c = \frac{24}{Re_p} \left( 1 + \frac{Re_p^2}{6} \right)$$

### Momentum equation of solid (particle) phase regarded as continuum (Eulerian approach)

$$\frac{d\mathbf{u}_p}{dt} = \frac{\mathbf{g}}{\rho_p} - \frac{1}{\rho_p} \text{grad}p + \frac{\mathbf{F}}{m_p} \quad \mathbf{u}_p \text{ [m/s] velocity field of particle phase}$$

$m_p$  [kg] mass of a particle

$\rho_p$  [kg/m<sup>3</sup>] density of particle phase,

$\mathbf{g}$  [N/kg] gravitational field of force

$\mathbf{F}_p$  [N] aerodynamic force acting on a particle

Pressure force can be neglected, so:  $\frac{d\mathbf{u}_p}{dt} = \frac{\mathbf{g}}{\rho_p} + \frac{\mathbf{F}}{m_p}$

By expressing  $\mathbf{F}_p$  and inserting it in the momentum equation of gas we obtain:

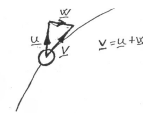
$$\frac{d\mathbf{v}}{dt} = \frac{\mathbf{g}}{\rho} - \frac{1}{\rho} \text{grad}p + \nu \Delta \mathbf{v} - \frac{n \cdot m_p}{\rho_s} \left( \frac{d\mathbf{u}_p}{dt} - \frac{\mathbf{g}}{\rho_p} \right) \quad \text{where } n \cdot m_p = c_p \text{ [kg/m}^3\text{] is solid concentration}$$

### Momentum equation for particles 1.

$$\frac{x^3 \pi}{6} \rho_p \frac{d\mathbf{u}}{dt} = \frac{x^3 \pi}{6} \rho_p \mathbf{g} + 3\pi \mu x \mathbf{w} \left| \frac{I_0}{v_0^2} \right.$$

Dimensionless equation:

$$\frac{d\frac{\mathbf{u}}{v_0}}{d\frac{t}{I_0/v_0}} = \frac{\mathbf{g} I_0}{v_0^2} + \frac{18\mu}{x^2 \rho_p} \frac{I_0}{v_0} \frac{\mathbf{w}}{v_0}$$



Settling of particle of  $\rho_p$  density in a gas of density  $\rho$ :

$$\frac{x^3 \pi}{6} \rho_p \mathbf{g} = \frac{x^3 \pi}{6} \rho \mathbf{g} + 3\pi \mu x \mathbf{w}_s \quad w_s \approx 0,04 (x [\mu\text{m}])^2$$

Settling velocity:  $w_s = \frac{x^2 (\rho_p - \rho) \mathbf{g}}{18\mu}$  Correction:  $w_{s,corr} = Cu w_s$

$Cu = 1 + \frac{2A\lambda}{x}$  Cunningham coefficient,  $A=1,4$ ,  $\lambda$  mean free path of gas molecules. (At room-temperature  $\lambda = 6.5 \cdot 10^{-2} \mu\text{m}$ ).

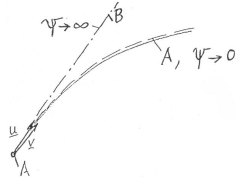
### Momentum equation for particles 2.

By neglecting the field of gravity  $\frac{du}{dt} = \frac{g l_0}{w_s v_0} \frac{w}{l_0 / v_0}$   
 and by introducing the inertia parameter:  $\psi = \frac{w_s v_0}{g l_0} = \frac{s}{l_0}$ ,  
 where  $s$  [m] stopping distance.

Dimensionless momentum equation for particles (' denotes dimensionless quantities):

$$\frac{d u'}{d t'} = \frac{1}{\psi} w' = \frac{1}{\psi} (v' - u')$$

Motion of particle in case of curved streamlines: due to their momentum particles move along path of larger radius of curvature than that of the streamline



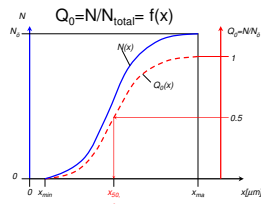
**isokinetic sampling**

$C_{s2} = C_1$  if  $v_{s2} = v_{1i}$

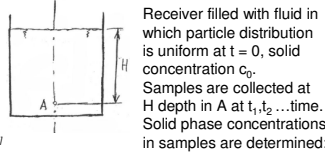
In case of non-isokinetic sampling the error of sampling ( $C/C_{s2}$ ) depends of the  $\Psi$  inertia parameter.

simple velocity null-pressure velocity and flow rate

### Application 1: determining of cumulative (undersize) distribution of particle assembly related to mass of particles by using sedimentation



$$Q_3 = M/M_{total} = f(x) = ?$$



Relation between  $t_s$  sampling time and size of particles  $x \geq x_1$  that – because of settling – are not present in the sample.  
 The ratio of  $M$  mass of particle fractions existing in the sample (and in and above point A) and the  $M_{total}$  mass of all fractions is equal to  $c_i/c_0$ . Knowing from sampling time  $x_1, x_2, \dots, x_i$  and  $c_i/c_0, c_2/c_0, \dots, c_i/c_0$   $Q_3 = M/M_{total} = f(x)$  can be constructed.  
 Problems: uniform distribution at  $t = 0$ , interaction of settling particles of different sizes.

$$x_i = \frac{H}{w_{si}} = \frac{18 \mu H}{x_i^2 (\rho_p - \rho_s) g}$$

$$x_i = \sqrt{\frac{18 \mu H}{t_i g (\rho_p - \rho_s)}}$$

### Separation of particles

Origin of technological aerosols, what to do in a given case?

Two steps of originating aerosol: Production of particles and their dispersion.  
 a) Production of particles: a1 intentional – a2 not-intentional  
 b) Dispersion of particles: b1 intentional – b2 not-intentional

3 variations:

- I. a1 – b1 Both production and dispersion are intentional: e.g. pulverized-coal fired boilers, use of catalytic agent in a gas to accelerate the reaction. No means for reducing the amount of gas and polluting particles so gas should be removed and cleaned or technology can be changed (use of gas instead of coal).
- II. a1 – b2 Production is intentional, dispersion is non-intentional: e.g. dispersion of cement powder at transport in production line. Dispersion can be reduced by reducing the particle velocity relative to air, and the air pollution in the neighborhood can be reduced by using hoods, casings, covers and exhaust and removal and cleaning of particle laden air (development of transport system),
- III. a2 – b2 Both production and dispersion are not intentional e.g. at explosion used in mining or at demolishing of buildings. Change of technology, use of water spray reducing the dispersion, covering the path of the particles, reducing the relative air velocity.

IV. a2 – b1 Not relevant

### Application 2: Measurement of mean dust concentration and dust mass rate in a duct.

Mean dust concentration  $\bar{c} [kg/m^3]$

$$\bar{c} = \frac{\int c v_{si} dA}{\int v_{si} dA} = \frac{\sum_{i=1}^n c_i v_{si} \Delta A_i}{\sum_{i=1}^n v_{si} \Delta A_i}$$

if  $\Delta A_i = const.$   $\bar{c} = \frac{\sum_{i=1}^n c_i v_{si}}{\sum_{i=1}^n v_{si}}$

Measured concentration:

$$\bar{c}_M = \frac{\sum_{i=1}^n \frac{d_{szi}^2 \pi}{4} v_{szi} \Delta t_i c_{szi}}{\sum_{i=1}^n \frac{d_{szi}^2 \pi}{4} v_{szi} \Delta t_i}$$

In  $i$ -th sampling point  $v_{szi}$  and  $c_{szi}$  are sampling velocity and dust concentration in the sampling probe of diameter  $d_{szi}$ ,  $\Delta t_i$  sampling period.

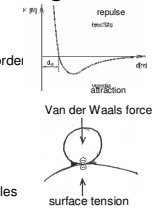
$$\bar{c}_M = \frac{\sum_{i=1}^n c_{si} v_{si}}{\sum_{i=1}^n v_{si}} = \bar{c} \text{ if } c_{szi} = C_i$$

Sampling head, shaft, particle separator (filter), measurement and control of flow rate (volume) of gas sample

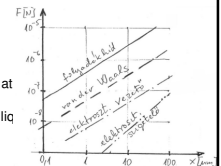
### Separation of particles from gas

Two steps of separation

1. The particles should be moved relative to the gas in order to establish contact with
  - solid surfaces,
  - fluid films, of drops much bigger than the particles,
  - other particles
 by utilizing effects moving the particles relative to the gas:
  - $\alpha$  inertia of particle
  - $\beta$  gravitational force
  - $\gamma$  diffusion caused by thermal agitation of gas molecules
  - $\delta$  electrostatic forces caused by charge of particles



2. Forces should be utilized to "stick" the particles
  - to each other,
  - to solid surfaces and
  - to surface of liquid (e.g. water films) by
    - A. Van der Waals force (attractive force between at
    - B. electrostatic attraction
    - C. surface tension (a property of the surface of a liq causes it to behave as an elastic membrane.



## Separators of significant role of inertia of particles

**Pre-separator louver of a filter Scrubber (washing towers)**

Labels: raw gas inflow, water spray, coarse dust particles, porous filter, dust collector bunker.

**Cyclone separator**

Labels: cleaned gas, dusty gas, row gas inflow.

**Venturi scrubber**

Labels: water, water drops in accelerating and decelerating gas flow.

$$F_c = \frac{x_h^3 \pi}{6} \rho_p \frac{v_{ii}^2}{r_i} = 3\pi \mu x_h v_{ii}$$

$$\bar{v}_{ii} \approx \frac{q_v}{2r_i \pi M} \quad v_{ii} = \frac{R_{bc} v_{bc}}{r_i}$$

$$\frac{v_{ii}^2 R_{bc}^2 x_h^2}{r_i r_i^2} \rho_p = v_{ii} = \frac{q_v}{2r_i \pi M} \quad x_h = \sqrt{\frac{9}{\pi} \frac{r_i}{R_{bc} v_{bc}}} \sqrt{\frac{\mu q_v}{\rho_p M}}$$

## Fibrous filters (woven fabrics, felts)

Labels:  $\alpha$ ,  $\beta$ ,  $\delta$ ,  $d_p$ ,  $d_f$ .

Mechanisms moving the particles relative to the gas towards fibres and causes collision of particles on fibre:

- curved streamlines: **inertia of particles**
- weight
- diffusion
- electrostatic forces
- blocking

Inertia (I) blocking (B) and diffusion (D)

$\alpha = 0,01 - 0,3$  solidity  
 $d_p \approx 10-30 \mu m$   
 $a/d_f \approx 1,6 - 9$

$\alpha = 0,1, a/d_f = 2,6, d_f = 20 \mu m, a = 52 \mu m$   
 Fabric width  $s = 4 \text{ mm} \Rightarrow 76$  rows of fibres

Domains of dominant effects influencing the collision Efficiency: T inertia, G weight, D diffusion, B blocking

Collision efficiency  $\varphi$  and collection efficiency  $\eta$  as function of filtration velocity  $\eta = \text{collected/colliding particles}$

Collision efficiency  $\varphi = \frac{\delta}{d_f}$

$\Psi = \frac{v_i w_v}{g d_f} = \frac{v_i \rho_p x^2}{18 \mu d_f}$  inertia parameter

Collision efficiency (inertia and blockage) as function of inertia parameter and Reynolds number

Collision efficiency as function of particle diameter and filtration velocity as parameter

Single fibre as particle separator

Change of pressure drop and filtration efficiency in time (separated dust mass)