<u>Technical Acoustics and Noise Control</u> *(lecture notes for self-learning)*

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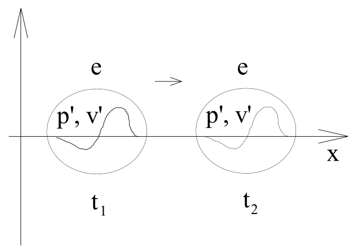
Lecture 8.

Content:

8.1. Energetics of acoustic waves, theoretic background (lecture notes)

During sound propagation simultaneously with the sound pressure, particle velocity and other variables propagation, the acoustic energy will be transported in the space as well. This physical fact can support with two explanation. If the base physical quantities related to the sound are propagating in the space, all of the resultant variables (e.g.: momentum, energy, …) must propagate too. For example, if the particle velocity disturbance moves from one point to another, the kinetic energy (half time mass time particle velocity square) will move away as well. For the other explanation let place a loudspeaker and a limited distant listener in a free space. Before switch on the speaker, in the approximate silence the test person's eardrums is in static rest. When switch on the loudspeaker, the pressure perturbations reach the eardrums, the flexible membrane, effected by the pressure difference, will displace a bit away. The product of the elementary pressure force and the displacement is an elementary work, done by the sound on the eardrum.



The transportation of the base and resultant variables during sound propagation

To know the energy transmitting ability of the sound is important from theoretic point of view, but it has several important practical application too. For example in engineering noise control, the hazardousness of a noise mostly characterised by the noise exposure (noise dose). The nose exposure is the product of the sound power enters the ear and the radiation time, the work, that exhaust the ear. So from noise control point of view, to know the energy transported by the sound is important.

To create the mathematic model of the energetics of sound wave, we will apply the same simplification, used in linear acoustics. The derivation concerns to one dimensional plane wave sound propagation. The direct use of the energy equations turns to complicate, so instead of the energy equation, let take the linear acoustic equation of motion and multiple both side of the equation with the particle velocity and equilibrium density,

$$\rho_0 v' \left(\frac{\partial v'}{\partial t}\right) = \rho_0 v' \left(\frac{-1}{\rho_0}\frac{\partial p'}{\partial x}\right)$$

After some mathematic operation,

$$\frac{\partial}{\partial t}\left(\rho_0 \frac{v'^2}{2}\right) = -\frac{\partial (p'v')}{\partial x} + p'\frac{\partial v'}{\partial x}$$

Let express the x derivative of the particle velocity from the linear acoustic continuity equation,

$$\frac{\partial v'}{\partial x} = \frac{-1}{\rho_0}\frac{\partial \rho'}{\partial t}$$

and move it in the final term of the modified equation of motion,

$$\frac{\partial}{\partial t}\left(\rho_0 \frac{v'^2}{2}\right) = -\frac{\partial (p'v')}{\partial x} + p'\frac{-1}{\rho_0}\frac{\partial \rho'}{\partial t}$$

Let express the density fluctuation from the formula of the seed of sound,

$$\rho' = \frac{p'}{a^2}$$

and move it in final term of the previous equation,

$$\frac{\partial}{\partial t}\left(\rho_0 \frac{v'^2}{2}\right) = -\frac{\partial (p'v')}{\partial x} + p'\frac{-1}{\rho_0 a^2}\frac{\partial p'}{\partial t}$$

After some mathematic operation, the equation to describe the energetic relation during plane sound wave propagation, the acoustic energy equation,

$$\frac{\partial}{\partial t}\left(\rho_0 \frac{v'^2}{2} + \frac{p'^2}{2\rho_0 a^2}\right) = -\frac{\partial (p'v')}{\partial x}$$

Comments:

- Let take a separate volume (V) of medium with an energy (E), the volumetric energy density (e),

$$e = \frac{E}{V} \quad \left[\frac{J}{m^3}\right]$$

- In the acoustic energy equation left hand side, inside the brackets the first term is the volumetric kinetic energy density (e_k), related to the particle velocity (v'),

$$e_k = \frac{E_k}{V} = \frac{\frac{1}{2}mv'^2}{V} = \frac{1}{2}\rho_0 v'^2$$

- The second term inside the bracket, left in the acoustic energy equation is volumetric potential energy density (e_p), creating by the pressure force, acting on the air. To see the details, let take the first law of thermodynamics for an elementary process,

$$de = dw + dq$$

Where the elementary change of the internal energy per unit volume (de), the elementary work per unit volume, done by the external forces on the system (dw) and elementary heat per unit volume supplied to the system (dq). During sound propagation, the elementary thermodynamic processes will take place without heat exchange (dq= 0 J/m^3), and let change the specific volume (v_V) to the reciprocal of the density,

$$\rho_0 c_v dT = \rho_0 p dv_V = -\rho_0 p d\left(\frac{1}{\rho}\right) = -\rho_0 p \frac{-1}{\rho^2} d\rho$$

Let apply the first law of thermodynamic for sound propagation (p= p'), change the total variables to the equilibrium one (ρ= ρ_0) and the elementary variables change to the time variant one (dρ= ρ'). The result of the integral, is the work per unit volume done by the external pressure force, with other words, the volumetric potential energy density (e_p), stored in the compressed "air-spring",

$$e_p = \rho_0 c_v T' = -\rho_0 \int p' \frac{-1}{\rho_0^2} d\rho' = \frac{1}{a^2 \rho_0} \int p' dp' = \frac{p'^2}{2\rho_0\, a^2}$$

- At the right side of the acoustic energy equation, the product of the sound pressure and the particle velocity is the instantaneous sound intensity (I'). The power is the work per unit time, and the work is the product of the force and displacement, with acoustic variables,

$$P' = \frac{\Delta W'}{\Delta t} = \frac{F' \Delta s'}{\Delta t} = F' \frac{\Delta s'}{\Delta t} = F' v'$$

The intensity is the effective power per unit area. Change the acoustic power to the product of the instantaneous force and particle velocity. Let notice, the ratio of the force and area is the pressure, the instantaneous sound intensity,

$$I' = \frac{P'}{A} = \frac{F' v'}{A} = p' v'$$

- Introducing the new variables (e_t, e_k, e_p and I'), the short form of the acoustic energy equation,

$$\frac{\partial e_t}{\partial t} = -\frac{\partial I'}{\partial x}\ , \qquad \text{where } e_t = e_k + e_p$$

- Let change the formula of the volumetric kinetic and potential energy density using the algebraic form of the linear acoustic equation of motion,

$$e_k = \frac{1}{2}\rho_0 v'^2 = \frac{1}{2}\rho_0 v' \frac{p'}{\rho_0 a} = \frac{p' v'}{2a} = \frac{I'}{2a}$$

$$e_p = \frac{p'^2}{2\rho_0\, a^2} = \frac{p' \rho_0 a v'}{2\rho_0\, a^2} = \frac{p' v'}{2a} = \frac{I'}{2a}$$

$$e_k = e_p$$

The result is acoustic energy equipartition phenomena. With words, during plane sound wave propagation the kinetic and potential energy density are equal to each other.

8.2. Test questions and solved problems

T.Q.1. Based on the equation of fluid mechanics, derive the relation among the kinetic and potential energy density and sound intensity. List the simplifications, explain the neglected terms in detail.

T.Q.2. Based on the acoustic energy equation let's prove the acoustic energy equipartition phenomena! Write example, why is important to know the energy, transported by sound waves!
