AM02

Műszaki áramlástan I.

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Ismétlés: mátrixműveletek

- Szorzás

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3$$

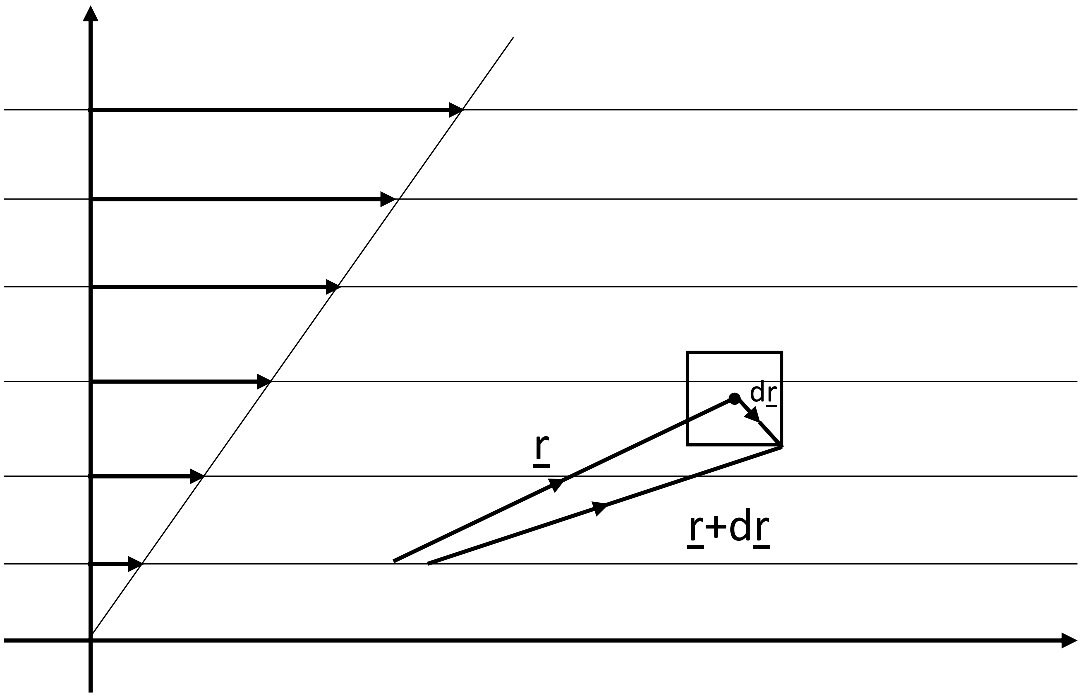
$$\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} x_1 \cdot a_1 + x_2 \cdot a_2 + x_3 \cdot a_3 \\ y_1 \cdot a_1 + y_2 \cdot a_2 + y_3 \cdot a_3 \\ z_1 \cdot a_1 + z_2 \cdot a_2 + z_3 \cdot a_3 \end{bmatrix}$$

- Keresztszorzás

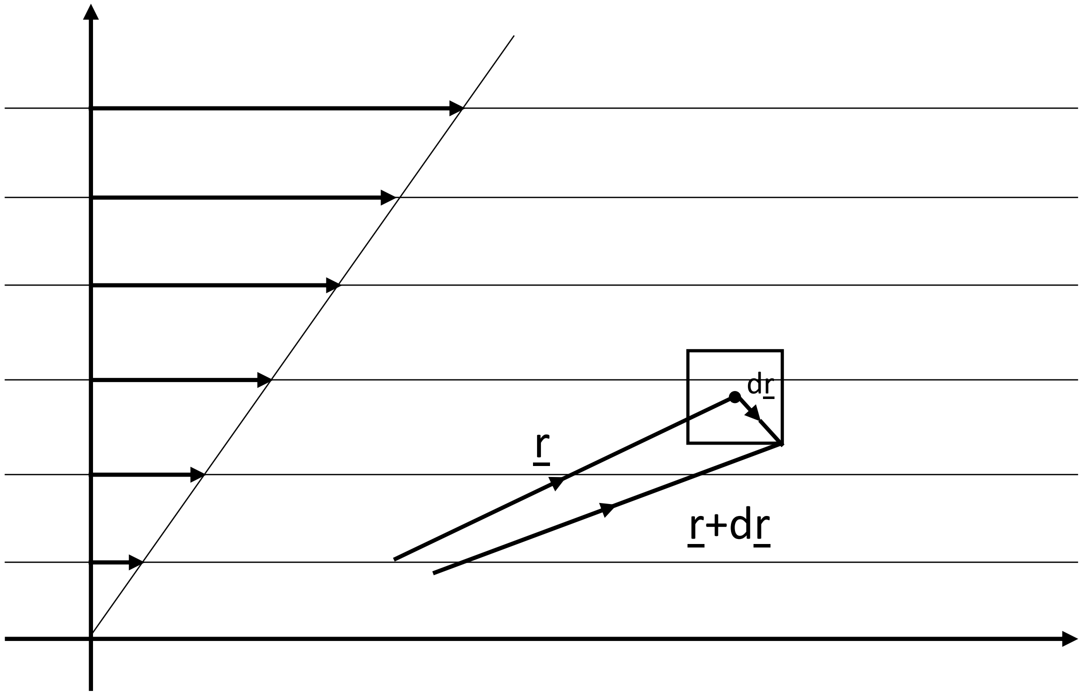
$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_2 \cdot b_3 - a_3 \cdot b_2 \\ a_3 \cdot b_1 - a_1 \cdot b_3 \\ a_1 \cdot b_2 - a_2 \cdot b_1 \end{bmatrix}$$

- Transzponálás: főátlóra tükrözés

Kis folyadékrész mozgása



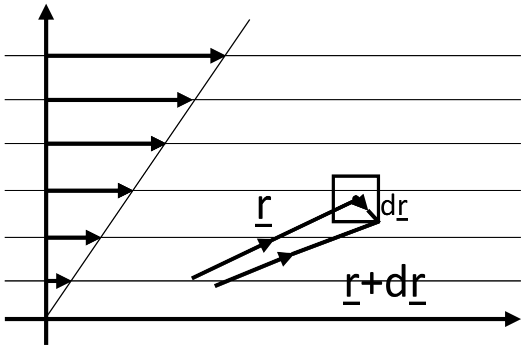
Hogyan mozog az elemi méretű folyadékhasáb?



$$\underline{v}(\underline{r} + d\underline{r}) \cong \underline{v}(\underline{r}) + \underline{\underline{D}} \cdot d\underline{r}$$

$$\underline{\underline{D}} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

Deriválttenzor



Deriválttenzor felbontása:

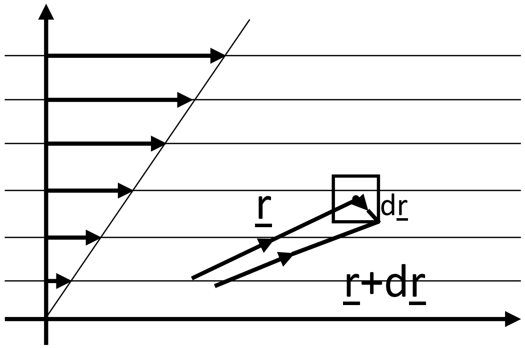
$$\underline{\underline{D}} = \underbrace{\frac{1}{2}\left(\underline{\underline{D}} + \underline{\underline{D}}^T\right)}_{\text{1. tag}} + \underbrace{\frac{1}{2}\left(\underline{\underline{D}} - \underline{\underline{D}}^T\right)}_{\text{2. tag}}$$

$$\underline{\underline{D}} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix} \qquad \underline{\underline{D}}^T = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_y}{\partial x} & \frac{\partial v_z}{\partial x} \\ \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial y} & \frac{\partial v_z}{\partial y} \\ \frac{\partial v_x}{\partial z} & \frac{\partial v_y}{\partial z} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

1. tag:

$$\underline{\underline{A}}_s = \frac{1}{2} \cdot \begin{bmatrix} \frac{\partial v_x}{\partial x} + \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial y} + \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} & \frac{\partial v_z}{\partial z} + \frac{\partial v_z}{\partial z} \end{bmatrix}$$

szimmetrikus



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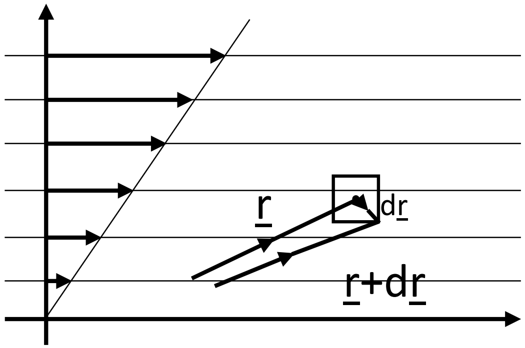
2. tag:

$$\underline{\underline{A}}_{\Omega} = \frac{1}{2} \cdot \begin{bmatrix} \frac{\partial v_x}{\partial x} - \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial y} - \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} & \frac{\partial v_z}{\partial z} - \frac{\partial v_z}{\partial z} \end{bmatrix}$$



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2. tag:

$$\underline{\underline{A}}_\Omega = \frac{1}{2} \cdot \begin{bmatrix} 0 & \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} & 0 & \frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} & 0 \end{bmatrix}$$

antiszimmetrikus

Ismétlés: rotáció

$$\underline{rot}\ \underline{v} = \underline{\nabla} \times \underline{v} = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \times \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \\ \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \end{bmatrix} = 2 \cdot \underline{\Omega}$$

vagyis:

$$\underline{rot}\ \underline{v} = \underline{\nabla} \times \underline{v} = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \times \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \\ \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \end{bmatrix} = 2 \cdot \underline{\Omega}$$

$$\underline{\underline{A}}_\Omega = \frac{1}{2} \cdot \begin{bmatrix} 0 & \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} & 0 & \frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} & 0 \end{bmatrix} = \frac{1}{2} \cdot \begin{bmatrix} 0 & -\left(\underline{rot}\ \underline{v}\right)_z & \left(\underline{rot}\ \underline{v}\right)_y \\ \left(\underline{rot}\ \underline{v}\right)_z & 0 & -\left(\underline{rot}\ \underline{v}\right)_x \\ -\left(\underline{rot}\ \underline{v}\right)_y & \left(\underline{rot}\ \underline{v}\right)_x & 0 \end{bmatrix}$$

$$\underline{\underline{A}}_{\Omega} = \frac{1}{2} \cdot \begin{bmatrix} 0 & \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} & 0 & \frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} & 0 \end{bmatrix} \qquad d\underline{r} = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}$$

$$\underline{\underline{A}}_{\Omega} \cdot d\underline{r} = \frac{1}{2} \cdot \begin{bmatrix} \cancel{0 \cdot dx} + \left(\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x}\right) \cdot dy + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) \cdot dz \\ \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) \cdot dx + \cancel{0 \cdot dy} + \left(\frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y}\right) \cdot dz \\ \left(\frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z}\right) \cdot dx + \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) \cdot dy + \cancel{0 \cdot dz} \end{bmatrix} =$$

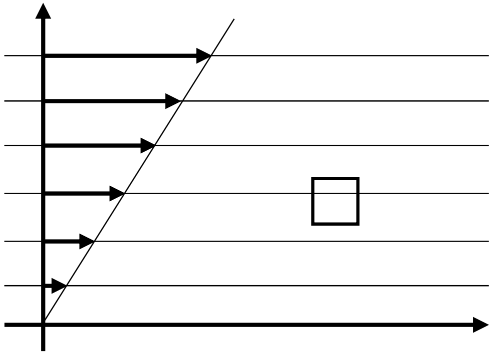
$$= \frac{1}{2} \cdot \begin{bmatrix} \left(\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x}\right) \cdot dy + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) \cdot dz \\ \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) \cdot dx + \left(\frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y}\right) \cdot dz \\ \left(\frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z}\right) \cdot dx + \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) \cdot dy \end{bmatrix}$$

$$\underline{rot}\ \underline{v} = \begin{bmatrix} \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \\ \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \end{bmatrix} \qquad d\underline{r} = \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}$$

$$\underline{rot}\ \underline{v} \times d\underline{r} = \begin{bmatrix} \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) dz - \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) dy \\ \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) dx - \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) dz \\ \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) dy - \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) dx \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) dz + \left(\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x}\right) dy \\ \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) dx + \left(\frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y}\right) dz \\ \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) dy + \left(\frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z}\right) dx \end{bmatrix}$$

$$\underline{\underline{A}}_{\Omega} \cdot d\underline{r} = \frac{1}{2} \cdot \begin{bmatrix} \left(\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x}\right) \cdot dy + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) \cdot dz \\ \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) \cdot dx + \left(\frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y}\right) \cdot dz \\ \left(\frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z}\right) \cdot dx + \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) \cdot dy \end{bmatrix} = \frac{1}{2} \cdot \underline{rot}\ \underline{v} \times d\underline{r} = \underline{\Omega} \times d\underline{r}$$

Gyakorlatban:

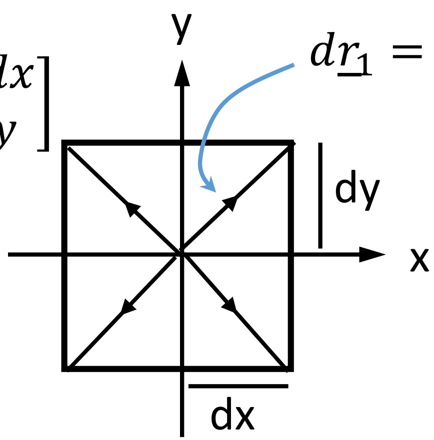


Tegyük fel, hogy a sebességtér: $\underline{v} = 4y \cdot \underline{i}$ vagyis $\underline{v} = v_x(y)$

Elemi hasáb:

$$d\underline{r_1} = dx \cdot \underline{i} + dy \cdot \underline{j} = \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$$d\underline{r_2} = \begin{bmatrix} -dx \\ dy \end{bmatrix}$$



Fejtsük ki:

A sebességtér: $\underline{v} = 4y \cdot \underline{i}$

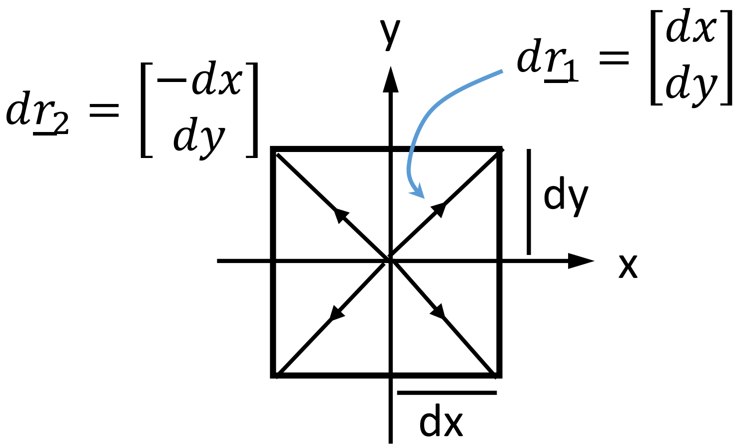
$$\underline{\underline{D}} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 0 & 0 \end{bmatrix}$$

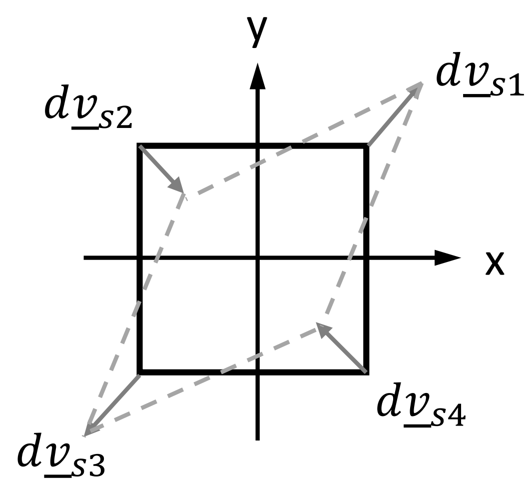
$$\underline{\underline{A}}_s = \frac{1}{2} \cdot \begin{bmatrix} \frac{\partial v_x}{\partial x} + \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} & \frac{\partial v_y}{\partial y} + \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} & \frac{\partial v_z}{\partial z} + \frac{\partial v_z}{\partial z} \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$$\underline{\underline{A}}_\Omega = \frac{1}{2} \cdot \begin{bmatrix} 0 & \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} & 0 & \frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

Deformáció:

A sebességtér: $\underline{v} = 4y \cdot \underline{i}$





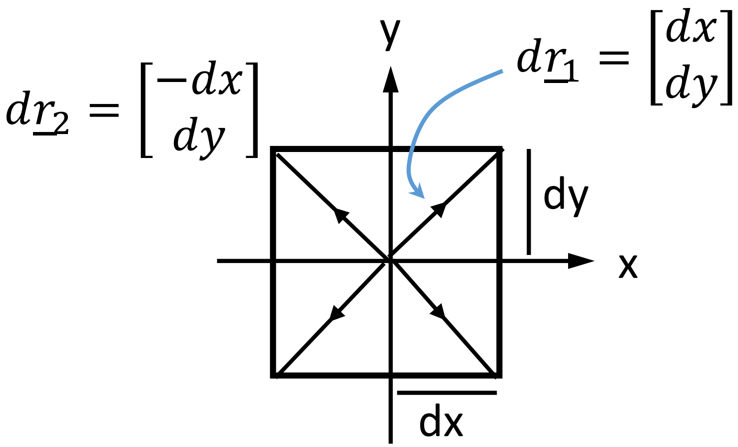
$$\underline{\underline{A}}_s = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

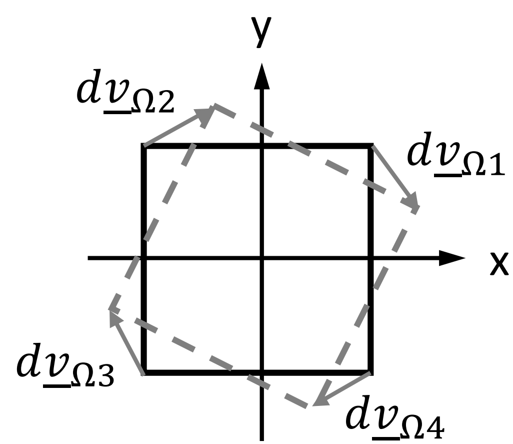
$$d\underline{v}_{s1} = \underline{\underline{A}}_s \cdot d\underline{r}_1 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} 2 \cdot dy \\ 2 \cdot dx \end{bmatrix}$$

$$d\underline{v}_{s2} = \underline{\underline{A}}_s \cdot d\underline{r}_2 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} -dx \\ dy \end{bmatrix} = \begin{bmatrix} 2 \cdot dy \\ -2 \cdot dx \end{bmatrix}$$

Rotáció:

A sebességtér: $\underline{v} = 4y \cdot \underline{i}$





$$\underline{\underline{A}}_{\Omega} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$d\underline{v}_{\Omega 1} = \underline{\underline{A}}_{\Omega} \cdot d\underline{r}_1 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \cdot \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} 2 \cdot dy \\ -2 \cdot dx \end{bmatrix}$$

$$d\underline{v}_{\Omega 2} = \underline{\underline{A}}_{\Omega} \cdot d\underline{r}_2 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \cdot \begin{bmatrix} -dx \\ dy \end{bmatrix} = \begin{bmatrix} 2 \cdot dy \\ 2 \cdot dx \end{bmatrix}$$

A teljes sebességváltozás:

$$\underline{v}(\underline{r}+d\underline{r}) \cong \underline{v}(\underline{r}) + \underline{\underline{D}} \cdot d\underline{r} = \underline{v}(\underline{r}) + \underline{\underline{A}}_s \cdot d\underline{r} + \underline{\underline{A}}_\Omega \cdot d\underline{r}$$

Párhuzamos elmozdulás

Deformálódás

Elfordulás

Ennek megfelelően:

$\underline{\underline{A}}_s$ Alakváltozási tenzor

$\underline{\underline{A}}_\Omega$ Örvénytenzor

Ha $\underline{\underline{D}}$ szimmetrikus, akkor $\underline{\underline{A}}_\Omega = 0$, vagyis $\underline{rot}\ \underline{v} = 0$, ekkor létezik a sebességtérnek potenciálja (Ψ), amire igaz, hogy $\underline{v} = \underline{grad}\Psi$

Kontinuitás

Zárt felületen a többletkiáramlás: $q_v = \int_A^{\cdot} \underline{v}\, d\underline{A}$ $\quad q_m = \int_A^{\cdot} \rho\, \underline{v}\, d\underline{A}$

Többletkiáramlás: a felületen belül (= a térfogatban) a sűrűség változik.

Ez a változás: $\int_V^{\cdot} \frac{\partial \rho}{\partial t} dV$

Ha a tömegáram pozitív, akkor több áramlik ki mint be: a térfogatban fogy a tömeg,

a sűrűség csökken: $\int_A^{\cdot} \rho\, \underline{v}\, d\underline{A} = -\int_V^{\cdot} \frac{\partial \rho}{\partial t} dV$

Gauss-Osztrogradszkij tétel alapján: $\int_A^{\cdot} \rho\, \underline{v}\, d\underline{A} = \int_V^{\cdot} div\, \rho \underline{v}\, dV = -\int_V^{\cdot} \frac{\partial \rho}{\partial t} dV$

Innen: $\int_V^{\cdot} div\, \rho \underline{v}\, dV + \int_V^{\cdot} \frac{\partial \rho}{\partial t} dV = 0 = \int_V^{\cdot} \left(div\, \rho \underline{v} + \frac{\partial \rho}{\partial t} \right) dV$

Kontinuitás

$$\int_V \left(div\ \rho\underline{v} + \frac{\partial\rho}{\partial t} \right) dV = 0$$

Ez akkor igaz, ha: $div\ \rho\underline{v} + \frac{\partial\rho}{\partial t} = 0$

A divergencia a deriválási szabályok szerint felbontva: $div\ \rho\underline{v} = \underline{v}\ \underline{grad}\ \rho + \rho\ div\underline{v}$

Átrendezve: $\frac{\partial\rho}{\partial t} + \underline{v}\ \underline{grad}\ \rho = -\rho\ div\underline{v}$

Lokális változás, az idő függvényében

Konvektív változás, azért változik mert A-ból B-be jut

Ha A-ból B-be eljut, akkor ρ_A-nak addigra ρ_B-vé kell változnia.

A változás értéke: $\underline{grad}\rho \cdot d\underline{s}$ ahol $d\underline{s} = \underline{v} \cdot dt$ ezért a változás: $\underline{grad}\rho \cdot \underline{v} \cdot dt$

Mivel egységnyi idő alatti változást vizsgáljuk, ezért marad a $\underline{v}\ \underline{grad}\ \rho$

Kontinuitás

Az előzőeknek megfelelően a sűrűség teljes változása: $$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \underline{v} \cdot \underline{grad}\rho$$

Ugyanez sebességtérre: $\underline{v} = \underline{v}(\underline{r}, t)$ $\quad \frac{d\underline{v}}{dt} = ?$

X irányú sebességkomponensre:

$$\frac{dv_x}{dt} = \frac{\partial v_x}{\partial t} + \underline{v} \cdot \underline{grad}\, v_x = \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}$$

$$\frac{d\underline{v}}{dt} = \begin{bmatrix} \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \end{bmatrix} = \frac{\partial \underline{v}}{\partial t} + \underline{\underline{D}}\, \underline{v}$$

Konvektív gyorsulás

Lokális gyorsulás

Kontinuitás

Példa: konfúzorban az áramlás 4 m/s-ról 6 m/s-ra nő egy 0.5 m-es szakaszon.

Gyorsulás: csak konvektív, csak X irányú komponens.

$$\underline{\underline{D}}\,\underline{v} = v_x \frac{\partial v_x}{\partial x}$$

$$\frac{\partial v_x}{\partial x} = \frac{6-4}{0.5} = 4\,\frac{1}{s}$$ Közelítés!

$$v_x = \frac{4+6}{2} = 5\,\frac{m}{s}$$ Ez is!

Eredmény: $a_{konv} = 5\,\frac{m}{s} \cdot 4\,\frac{1}{s} = 20\,\frac{m}{s^2}$