<u>Technical Acoustics and Noise Control</u> *(lecture notes)*

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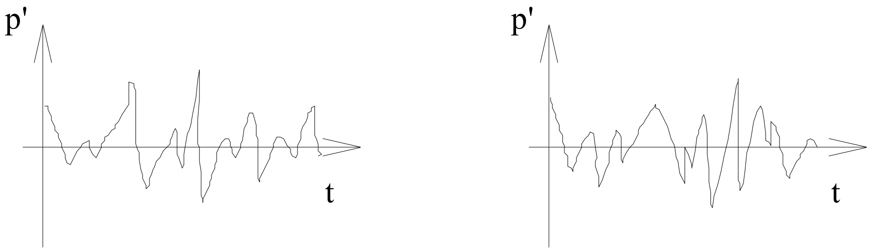
Content:

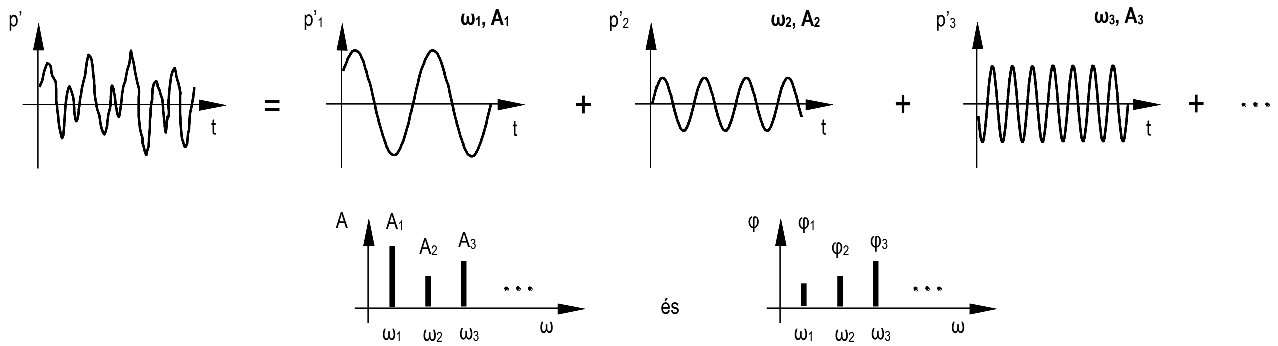
9.1. Harmonic analysis and sound spectra (lecture notes)

First let speak about the motivation, why is important the harmonic analysis and sound spectra in the noise engineering practise. In a sound field, point by point the time history of the acoustic variables include all of the important information. Despite of this, comparing the time functions of absolutely different sound effects, often the shape of the functions are very similar, and we cannot make difference between the sounds, based on their time functions. Left hand side of the drawing we can see sound pressure time variation 30m far from a tracked machine, right hand side we can see similar data, but in a music hall, during the performance of a chamber orchestra. At first sight there is no characteristic difference between the diagrams, but the subjective impressions are really different. In spite of the time function contain a lot important information about the sound field, the direct use in the acoustic engineering problem analysis and design is limited.



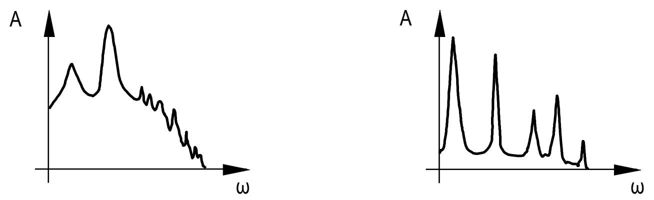
Sound pressure as a function of time 30m far from tracked machine (left) and at a performance of a chamber orchestra (right)

So we need a method that can clear the information hidden in the time function. Important, that the acoustic variables (with periodic or not periodic character) alter around the ambient equilibrium pressure. It looks promising to find simple, mathematic base function, that describes a variation around a mean value. We suspect, that the behaviour of the sound concerning the generation, propagation, decay and sensation of it, will different at long and short cycles. So decomposing the original time function, the missing information will turn out. There are several periodic function in mathematics, but most of them are problematic (nonanalytic, discontinuous, …). The sinus and cosine functions satisfies all conditions. Further increase the importance of sinus and cosine functions, the fact, that the natural vibration of finite size flexible structures are harmonic vibrations or the composition of this (described with sine or cosine) and the harmonic vibrations will radiate harmonic waves. Summarising it looks useful to decompose the original time function to different time of period and amplitude harmonic components. The next picture symbolises the graphic decomposition.



The graphic decomposition of the time function to harmonic component

The sinus and cosine are periodic functions, so their time histories contain a lot of redundant information. To rebuild an arbitrary time function it enough to know the amplitude and initial phase of the needed harmonic components. The frequency amplitude and frequency initial phase functions are called amplitude and phase spectrum of the original time function. In mechanical engineering noise control often it is enough to know the amplitude spectrum, the phase spectrum has more application in the mechanical engineering fault diagnostics.



Sound pressure amplitudes as a function of angular frequency 30m far from a tracked working machine (left) and at a chamber orchestra performance (right)

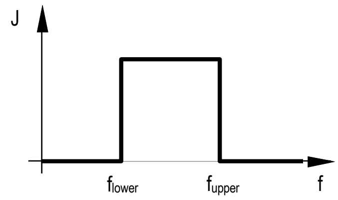
Left hand side on the schematic drawing, the broad band frequency distribution relates to disturbing noise effect. The high amplitude low frequency component is due to periodic exhaust of the diesel engine, the components at upper frequency range belongs to creaking track noise. The right hand side of the picture shows big amplitude discrete frequency component, and low amplitude broad band noise. This can relates to solid or fluid mechanic resonant sound generation. When calculate the ratio of upper and base frequency components, it will turn out that it should be a musical sound.

The harmonic components of an optional periodic time function can calculate with the Fourier-series of the function. The harmonic decomposition of an optional aperiodic time function can calculate with the Fourier-integral. In general case the spectrum of a periodic function will consist of unlimited number discrete frequency components (different frequency values), the non-periodic functions have continuous spectrums.

Harmonic analysis in measurement practise

Today to determine a spectrum of a time function we will use digital signal processing. The first step of the digital signal processing is the digital sampling. The digital sampling completed by a special electronic unit (called digital data collector) that will measure the electronic signal (e.g. microphone voltage) rapidly, periodically several time and store the measured values. From the digital sample sequence, the digital spectrum will be available with DFT (Discrete Fourier Transform), or with the fastened version FFT (Fast Fourier Transform) algorithm. Based on DFT, FFT, lot of other signal processing method is known, as STFT (Short Time Fourier Transform), order tracking, and envelop detection, that are wide spread in the mechanical engineering noise and vibration control and fault diagnostics.

Because the application advantages, the harmonic analysis was deeply involved to solve acoustic problems much more before the grow up of the digital signal processing. In this early time the harmonic decomposition was completed by the use a series of a band pass filters. The band pass filter admit the signal (J) only between a given lower and upper frequency.



Theoretic band pass filter frequency characteristic

If we build up a sequence of filter on the complete frequency range of interest, where the upper boarder frequency of the lower band is equal with the lower boarder frequency of the upper band, and switching on the filters following each other, we can scan the complete frequency range and determine a band spectrum. The magnitude of the band with determines the frequency resolution of the spectrum. Based on the relation of upper and lower boarder frequencies, the band pass filter sets can classify as constant absolute and constant relative band width types.

Constant absolute band width: $f_{upper} - f_{lower} = const.$

Constant relative band width: $\frac{f_{upper}}{f_{lower}} = const.$

In the case of constant absolute band width the frequency resolution is optional, a special value is the 1Hz. At constant relative band width often used resolutions are the octave and the one-third octave bands. Frequencies to determine the octave,

$$f_{oct\ upper} = 2\ f_{oct\ lower} \qquad f_{oct\ middle} = \sqrt{f_{oct\ lower}\ f_{oct\ upper}} = \sqrt{2}\ f_{oct\ lower} = f_{oct\ upper}/\sqrt{2}$$

Standardised octave band middle frequencies: … 31,5 63 125 250 500 1k 2k 4k 8k 16k … [Hz].

The advantage of the octave that to cover the complete audible frequency range, only 10 value is enough. The disadvantage is, that mostly in the upper frequency range the resolution is bad. This problem will partly handled by the one-third octave band. Frequencies to determine the one-third octave,

$$f_{third\ upper} = \sqrt[3]{2}\ f_{third\ lower}$$

$$f_{third\ middle} = \sqrt{f_{third\ lower}\ f_{third\ upper}} = \sqrt[6]{2}\ f_{third\ lower} = f_{third\ upper}/\sqrt[6]{2}$$

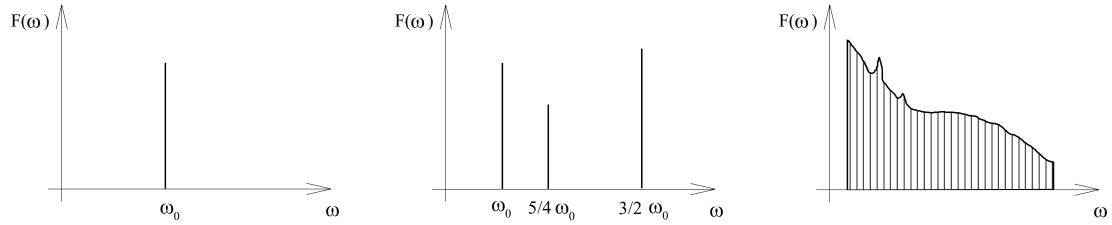
Standardised one-third octave band middle frequencies: … 31,5 40 50 63 80 100 125 160 200 … [Hz].

Classifying sound waves using sound spectra

Pure tone: The pure tone is a sound wave, consists of only one ω angular frequency harmonic component. The spectrum of the pure tone is one vertical line at ω angular frequency.

Musical sound: A consonant composition of pure tones is musical sound. The condition of the consonance when two or more pure tone exist in the same place and same time together is, that ratios of frequencies have to be equal with prescribed values.

Noise: A dissonant composition of pure tones is noise. The spectrum of a noise is often continuous.



Sound spectrums: pure tone (left), musical sound (major triad, mid), noise (fan noise, right)

Pitch and colour of sound

Pitch and colour are subjective phenomena of sound. Pitch is the subjective sensation of the sound frequency. The colour of a sound depends on the number and structure of the upper frequency components, accompanied to the base frequency. To sense the colour of a sound gives the possibility to distinguish the tone of a clarinet and violin play at the same frequency, if we do not see the players.

Sounds for acoustic testing

Pure tone: One of the most important test sound in acoustics is the pure tone. Advantage: big signal to noise ratio, disadvantage: long measurement time on the complete audible range.

Swept sinus: The swept sinus is a pure tone sound with a continuously varying frequency on the test range. Advantages: big signal to noise ratio and short measurement time on the complete audible range, disadvantage: sophisticated test apparatus requirement.

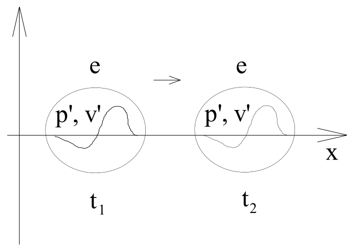
White noise: White noise is a random sound. In a long time period average white noise spectrum the amplitudes of the frequency components are identical. The spectrum of the white noise is a horizontal line. The name of white noise originated from optics, where the white colour was the mixture of different frequency components. A white noise is generated, when the air is released from a pressurised reservoir. Advantages: short measurement time on the complete audible range, disadvantage: low signal to noise ratio.

Pink (rose): The spectrum of a pink noise can characterise with increasing frequency a decreasing amplitude. The pink is the mixture of the broad band white and low frequency red. In constant relative bandwidth resolution the spectrum of the rose noise is a horizontal line. Comparing to the white noise the advantage of the pink noise is the higher signal to noise ratio in the usually critical low frequency range.

Impulsive noise: The short duration (~10ms, or smaller) test sounds allow to investigate the sound propagation process in time, the reverberation effects and the phase behaviour in acoustic systems.

9.2. Energetics of acoustic waves (theoretic background, lecture notes)

During sound propagation simultaneously with the sound pressure, particle velocity and other variables propagation, the acoustic energy will be transported in the space as well. This physical fact can support with two explanation. If the base physical quantities related to the sound are propagating in the space, all of the resultant variables (e.g.: momentum, energy, …) must propagate too. For example, if the particle velocity disturbance moves from one point to another, the kinetic energy (half time mass time particle velocity square) will move away as well. For the other explanation let place a loudspeaker and a limited distant listener in a free space. Before switch on the speaker, in the approximate silence the test person's eardrums is in static rest. When switch on the loudspeaker, the pressure perturbations reach the eardrums, the flexible membrane, effected by the pressure difference, will displace a bit away. The product of the elementary pressure force and the displacement is an elementary work, done by the sound on the eardrum.



The transportation of the base and resultant variables during sound propagation

To know the energy transmitting ability of the sound is important from theoretic point of view, but it has several important practical application too. For example in engineering noise control, the hazardousness of a noise mostly characterised by the noise exposure (noise dose). The nose exposure is the product of the sound power enters the ear and the radiation time, the work, that exhaust the ear. So from noise control point of view, to know the energy transported by the sound is important.

To create the mathematic model of the energetics of sound wave, we will apply the same simplification, used in linear acoustics. The derivation concerns to one dimensional plane wave sound propagation. The direct use of the energy equations turns to complicate, so instead of the energy equation, let take the linear acoustic equation of motion and multiple both side of the equation with the particle velocity and equilibrium density,

$$\rho_0 v'\left(\frac{\partial v'}{\partial t}\right) = \rho_0 v'\left(\frac{-1}{\rho_0}\frac{\partial p'}{\partial x}\right)$$

After some mathematic operation,

$$\frac{\partial}{\partial t}\left(\rho_0 \frac{v'^2}{2}\right) = -\frac{\partial(p'v')}{\partial x} + p'\frac{\partial v'}{\partial x}$$

Let express the x derivative of the particle velocity from the linear acoustic continuity equation,

$$\frac{\partial v'}{\partial x} = \frac{-1}{\rho_0}\frac{\partial \rho'}{\partial t}$$

and move it in the final term of the modified equation of motion,

$$\frac{\partial}{\partial t}\left(\rho_0 \frac{v'^2}{2}\right) = -\frac{\partial(p'v')}{\partial x} + p'\frac{-1}{\rho_0}\frac{\partial \rho'}{\partial t}$$

Let express the density fluctuation from the formula of the seed of sound,

$$\rho' = \frac{p'}{a^2}$$

and move it in final term of the previous equation,

$$\frac{\partial}{\partial t}\left(\rho_0 \frac{v'^2}{2}\right) = -\frac{\partial(p'v')}{\partial x} + p'\frac{-1}{\rho_0 a^2}\frac{\partial p'}{\partial t}$$

After some mathematic operation, the equation to describe the energetic relation during plane sound wave propagation, the acoustic energy equation,

$$\frac{\partial}{\partial t}\left(\rho_0 \frac{v'^2}{2} + \frac{p'^2}{2\rho_0 a^2}\right) = -\frac{\partial (p'v')}{\partial x}$$

Comments:

- Let take a separate volume (V) of medium with an energy (E), the volumetric energy density (e),

$$e = \frac{E}{V} \quad \left[\frac{J}{m^3}\right]$$

- In the acoustic energy equation left hand side, inside the brackets the first term is the volumetric kinetic energy density (e_k), related to the particle velocity (v'),

$$e_k = \frac{E_k}{V} = \frac{\frac{1}{2} m v'^2}{V} = \frac{1}{2}\rho_0 v'^2$$

- The second term inside the bracket, left in the acoustic energy equation is volumetric potential energy density (e_p), creating by the pressure force, acting on the air. To see the details, let take the first low of thermodynamics for an elementary process,

$$de = dw + dq$$

Where the elementary change of the internal energy per unit volume (de), the elementary work per unit volume, done by the external forces on the system (dw) and elementary heat per unit volume supplied to the system (dq). During sound propagation, the elementary thermodynamic processes will take place without heat exchange (dq= 0 J/m^3), and let change the specific volume (v_V) to the reciprocal of the density,

$$\rho_0 c_v dT = \rho_0 p dv_V = -\rho_0 p d\left(\frac{1}{\rho}\right) = -\rho_0 p \frac{-1}{\rho^2} d\rho$$

Let apply the first low of thermodynamic for sound propagation (p= p'), change the total variables to the equilibrium one ($\rho = \rho_0$) and the elementary variables change to the time variant one ($d\rho = \rho'$). The result of the integral, is the work per unit volume done by the external pressure force, with other words, the volumetric potential energy density (e_p), stored in the compressed “air-spring”,

$$e_p = \rho_0 c_v T' = -\rho_0 \int p' \frac{-1}{\rho_0^2} d\rho' = \frac{1}{a^2 \rho_0} \int p' dp' = \frac{p'^2}{2\rho_0\, a^2}$$

- At the right side of the acoustic energy equation, the product of the sound pressure and the particle velocity is the instantaneous sound intensity (I'). The power is the work per unit time, and the work is the product of the force and displacement, with acoustic variables,

$$P' = \frac{\Delta W'}{\Delta t} = \frac{F' \Delta s'}{\Delta t} = F' \frac{\Delta s'}{\Delta t} = F' v'$$

The intensity is the effective power per unit area. Change the acoustic power to the product of the instantaneous force and particle velocity. Let notice, the ratio of the force and area is the pressure, the instantaneous sound intensity,

$$I' = \frac{P'}{A} = \frac{F'v'}{A} = p'v'$$

- Introducing the new variables (e_t, e_k, e_p and I'), the short form of the acoustic energy equation,

$$\frac{\partial e_t}{\partial t} = -\frac{\partial I\prime}{\partial x}\ , \qquad \text{where } e_t = e_k + e_p$$

- Let change the formula of the volumetric kinetic and potential energy density using the algebraic form of the linear acoustic equation of motion,

$$e_k = \frac{1}{2}\rho_0 v'^2 = \frac{1}{2}\rho_0 v' \frac{p'}{\rho_0 a} = \frac{p'v'}{2a} = \frac{I'}{2a}$$

$$e_p = \frac{p'^2}{2\rho_0\ a^2} = \frac{p'\rho_0 a v'}{2\rho_0\ a^2} = \frac{p'v'}{2a} = \frac{I'}{2a}$$

$$e_k = e_p$$

The result is acoustic energy equipartition phenomena. With words, during plane sound wave propagation the kinetic and potential energy density are equal to each other.

9.3. Test questions and solved problems

T.Q.1. Define the octave and the one-third octave, and put down the related middle frequencies!

T.Q.2. Based on the equation of fluid mechanics, derive the relation among the kinetic and potential energy density and sound intensity. List the simplifications, explain the neglected terms in detail.

T.Q.3. Based on the acoustic energy equation let's prove the acoustic energy equipartition phenomena! Write example, why is important to know the energy, transported by sound waves!
